RESOURCE LETTER

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This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course content in specified fields. [The letter E after an item indicates elementary level or material of general interest to persons becoming informed in the field. The letter I, for intermediate level, indicates material of somewhat more specialized nature; and the letter A indicates rather specialized or advanced material.] No Resource Letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, 116 Church Street SE, University of Minnesota, Minneapolis, MN 55455.

Resource Letter: MesP-1: Mesoscopic physics

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This Resource Letter provides a guide to the literature on mesoscopic electron physics of solids. Journal articles, conference proceedings, and books are cited for the following topics: conductance fluctuations in disordered and quantum-chaotic systems, conductance quantization, conduction of a Luttinger liquid, electron noise in mesoscopic devices, mesoscopic superconductivity, electron–electron interactions in mesoscopic systems and the Coulomb blockade phenomenon, and Kondo effect in quantum dots.

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I. INTRODUCTION

In most of the commonly used conductors the electric current is carried by electrons. Electrons are elementary particles that have a discrete charge. The electron charge was measured in the seminal experiments of Millikan almost a century ago. Diffraction experiments performed at the dawn of the era of quantum physics also have demonstrated convincingly that an electron propagates as a wave. The wavelength depends on the electron energy, and in some cases may be viewed as the typical ‘‘size’’ of an electron. The electron energy varies in the range of a few electron-volts (eV) in a typical metal to a few milli-electron-volts (meV) in semiconductor structures used in modern-day electronic devices. The corresponding ‘‘sizes’’ range from a few angstroms to a few hundred of angstroms. The particle and wave properties of individual electrons are hardly important in usual electrical wires supplying electricity, say, to light fixtures in a room. The wire width is about 10 million times the size of an electron. After flipping a switch to light a 100-W bulb, we are sending electrons at a rate of about 10^{19} particles/s through the wires. Electrons flow through wires like liquid, much like water flows through plumbing pipes. The quality of a piece of wire as a conductor is characterized by its conductance, which is the current passing through the wire divided by the voltage bias applied between the wire ends. The conductance is inversely proportional to the wire length, and scales linearly with its cross-sectional area. The proportionality coefficient, or conductivity, characterizes the material the wire is made of, but not the shape of the wire. One may ask: What happens with this simple scaling law for the conductance when one makes a wire thinner and shorter? It turns out that the scaling law breaks down when the wire size is small enough to allow coherent propagation of an electron across it. For such small objects, conductance reflects not only the average properties of the material, but the specific shape, and even the specific distribution of obstacles for electron motion in a given sample. The fairly new field of physics studying such conductors, tiny enough to allow for coherent electron propagation, but still consisting of a huge (typically about 10^{10}) number of atoms, is called mesoscopic physics (meso here stands for the mentioned intermediate length scale). Conductance strongly deviates from the predictions of classical physics for wires with the width of the order of the electron ‘‘size.’’ It turns out that the conductance of a wire in these conditions may become quantized in universal, material-independent units of e^2/h, which are composed from two fundamental constants: electron charge e and Planck constant h. A wire with quantized conductance operates as an electron waveguide.

Extending the above example of a circuit involving a wire, a bulb, and a switch, one may also ask about the operation of a switch when it is made tiny enough. Can a switch dispense electrons one by one? It turns out that this is indeed possible owing to the so-called Coulomb blockade effect. The energy associated with charging of a small conductor by a single electron can be estimated (in CGS units) as e^2/L, where L is the linear size of that conductor. For a submicron size L, and at liquid helium temperatures ($T \lesssim 4$ K), this energy exceeds the thermal energy k_BT, and statistical physics allows for a well-defined number of electrons on such a tiny conducting island (here k_B is the Boltzmann constant). Essentially, physics of the charging of an isolated island by discrete electrons is classical electrostatics.

Experimenting with tiny wires and tiny conducting islands where, respectively, the quantum wave and classical particle properties of electrons become evident, accelerated during the 1990s because of the rapid advances in semiconductor nanotechnology. It is now possible to produce small devices that, by adjustment of several control knobs, may ''morph'' continuously from an electron waveguide to an isolated island—a quantum dot carrying a few electrons. These new experiments raise a number of interesting theoretical questions about the fundamental properties of mesoscopic systems, for which simultaneously the particle and wave properties of electrons are important.

The field of mesoscopic physics is rapidly evolving, and the list of topics mentioned is far from being complete and exhaustive. In the presentation of literature, an emphasis is placed on the available review articles.

II. JOURNALS, BOOKS, CONFERENCE PROCEEDINGS, AND WEB RESOURCE COVERING A VARIETY OF TOPICS IN MESOSCOPIC PHYSICS

A. Journals

A significant portion of the recent original papers in mesoscopic physics is published in general science journals:
Nature
Science
Many of the most important results are published in physics journals, especially in:
Physical Review Letters
Physical Review B (Condensed Matter)
Europhysics Letters
Journal of Physics C (Solid State Physics) continued since 1989 as
Journal of Physics (Condensed Matter)
Solid State Communications
JETP Letters (this is a translation from Russian of *Pis'ma v ZhETF*)
Soviet Physics–JETP (this is a translation from Russian of *Zhurnal Eksperimental'noi i Teoreticheskoi fiziki*)
A number of reviews and popular articles related to mesoscopic physics appeared, respectively, in
Reviews of Modern Physics
Physics Reports
and in
Physics Today
Physics World

B. Major compilations and texts

Of the nine books listed below, the first seven are collections of review articles. These seven volumes are only a few of the many published between 1991 and 1999. The selection gives an idea about the evolution of research subjects in the field of mesoscopic physics. The other two books are the available texts. Although both texts are written by theorists, they provide different perspectives of the field.

1. **Mesoscopic Phenomena in Solids**, edited by B.L. Altshuler, P.A. Lee, and R.A. Webb (North-Holland, Amsterdam, 1991). (I)
2. ''Quantum Transport in Semiconductor Nanostructures,'' C.W.J. Beenakker and H. van Houten, in **Solid State Physics**, edited by H. Ehrenreich and D. Turnbull **44**, 1–228 (1991). (I)
3. **Single Charge Tunneling (Coulomb Blockade Phenomena in Nanostructures)**, edited by H. Grabert and M.H. Devoret (Plenum, New York, 1992). (I)
4. **Mesoscopic Quantum Physics**, edited by E. Akkermans, G. Montambaux, J.-L. Pichard, and J. Zinn-Justin (Elsevier, Amsterdam, 1995). (I)
5. **Mesoscopic Electron Transport**, edited by L.L. Sohn, L.P. Kouwenhoven, and G. Schön (Kluwer Academic, Dordrecht, 1997). (I)
6. **Transport in Nanostructures**, edited by D.K. Ferry and S.M. Goodnick (Cambridge U.P., Cambridge, 1997). (I)
7. **Nanotechnology**, edited by G. Timp (Springer, New York, 1999). (I)
8. **Introduction to Mesoscopic Physics**, Yoseph Imry (Oxford U.P., Oxford, 1997). (I)
9. **Electronic Transport in Mesoscopic Systems**, Supriyo Datta (Cambridge U.P., Cambridge, 1995). (I)

C. Conference proceedings

Mesoscopic physics is represented in many regular international and national meetings of a broader scope. Over the past decade there also were numerous specialized meetings devoted solely to various aspects of mesoscopic physics. Many such meetings have published proceedings, examples are Refs. 3–5 above.

In this section, only the latest proceedings of the two major regular international conferences, which traditionally attract a significant portion of the mesoscopic physics community, are quoted:

10. **International Conference: Electronic Properties of Two-Dimensional Systems (EP2DS)**. Meets every two years. Proceedings of the latest conference, EP2DS-13 held in 1999, were published in **Physica E: Low-dimensional Systems and Nanostructures 6** (2000). (I)

The leading themes of this conference series evolve around the electronic properties of semiconductor heterostructures. The majority of the recent experiments in mesoscopic physics is performed on devices based on heterostructures. That makes these conferences a natural forum for discussing the advances in mesoscopic semiconductor physics.

11. **International Conference: Low-Temperature Physics (LT)**. Meets every three years. Proceedings of the latest conference, LT-22 held in 1999, were published in Phys. B **280** (2000). (I)

These international meetings attract a large group of researchers working in the low-temperature physics of normal and superconducting metals. Seminal experiments in mesoscopic physics were performed on submicron metallic wires and grains in the mid-1980s. Later, a number of interesting phenomena were discovered in small-size superconductors and contacts of superconductors with normal metals. A number of topics in mesoscopic physics, such as Coulomb blockade and mesoscopic superconductivity, are represented at the LT conferences.

D. Website

Many papers become available prior to a journal publication through the e-Print archive maintained by the Los Alamos National Laboratory. The archive has a separate subject class, Mesoscopic Systems and Quantum Hall Effect, at the following address:

12. http://www.arXiv.org/list/cond-mat.mes-hall/recent (A)

III. UNIVERSAL CONDUCTANCE FLUCTUATIONS (UCF) IN DISORDERED CONDUCTORS

Phase coherence of the quantum states of conduction electrons in a disordered metal can be preserved over 50–5000 nm. These ''mesoscopic'' distances greatly exceed a single lattice spacing (about 0.5 nm), but still are much smaller than the size of a conventional macroscopic device. The coherence can produce quantum-interference effects in electrical resistance of a mesoscopic metallic sample. The most striking manifestation of the interference consists in the variation of the sample resistance placed in a fairly weak magnetic field. The pattern of seemingly random variation is specific for a given sample, forming its ''magneto-fingerprint.'' The characteristic amplitude of the conductance (i.e., inverse resistance) variation does not depend on a specific mesoscopic device, and is of the order e^2/h, where e and h are the electron charge and the Planck constant, respectively. The random nature of the variations and the universality of their amplitude gave the name to the phenomenon—universal conductance fluctuations, or UCF.

A popular description of the UCF effect can be found in two articles,

13. ''Disordered electronic systems,'' B.L. Altshuler and P.A. Lee, Phys. Today **41**, 36–44 (1988). (E)

14. ''Quantum interference fluctuations in disordered metals,'' R.A. Webb and S. Washburn, Phys. Today **41**, 46–53 (1988). (E)

Conductance fluctuations in metal Au and AuPd wires were reported in:

15. ''Magnetoresistance of small quasi-one-dimensional, normal metal rings and lines,'' C.P. Umbach, S. Washburn, R.B. Laibowitz, and R.A. Webb, Phys. Rev. B **30**, 4048–4051 (1984). (I)

Theoretical understanding of the UCF was developed around 1985. The universality of the conductance fluctuations was predicted in papers:

16. ''Fluctuations in the extrinsic conductivity of disordered conductors,'' B.L. Altshuler, JETP Lett. **41**, 648–651 (1985) [Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530–533 (1985)]. (A)

17. ''Universal conductance fluctuations in metals,'' P.A. Lee and A.D. Stone, Phys. Rev. Lett. **55**, 1622–1625 (1985). (A)

These fluctuations also were seen in numerical simulations,

18. ''Magnetoresistance fluctuations in mesoscopic wires and rings,'' A.D. Stone, Phys. Rev. Lett. **54**, 2692–2695 (1985). (I)

After the publication of the Altshuler–Lee–Stone theory, a consensus rapidly developed that it properly accounts for the conductance fluctuations observed in experiments with mesoscopic metallic and semiconducting wires. A simplified theoretical description and further references to a number of experimental papers that appeared in the late 1980s can be found in the review of C. W. J. Beenakker and H. van Houten, see Ref. 2 above. The authors of the experimental discovery of UCF in mesoscopic metallic wires and rings review this effect in:

19. ''Quantum transport in small disordered samples from the diffusive to the ballistic regime,'' S. Washburn and R.A. Webb, Rep. Prog. Phys. **55**, 1311–1383 (1992). (I)

Along with the dc conductance, other transport and thermodynamic characteristics of small conductors also exhibit mesoscopic fluctuations. I give here only three further examples. First, mesoscopic fluctuations create sample-specific asymmetry of a small junction, which leads to a rectification of an ac current passing through it,

20. ''Mesoscopic photovoltaic effect in microjunctions,'' V.I. Fal'ko and D.E. Khmel'nitskii, Sov. Phys.-JETP **68**, 186–191 (1989) [Zh. Eksp. Teor. Fiz. **95**, 328–337 (1989)]. (I)

Second, a static magnetic flux threading a small metallic ring induces its mesoscopic magnetization,

21. ''Magnetic response of a single, isolated gold loop,'' V. Chandrasekhar, R.A. Webb, M.J. Brady, M.B. Ketchen, M.J. Gallagher, and A. Kleinsasser, Phys. Rev. Lett. **67**, 3578–3581 (1991). (I)

This magnetization can be viewed as the result of a sample-specific *persistent current* flowing in the ring, see the text by Y. Imry, Ref. 7. The third example is related to the so-called drag effect. A current passing through one of the closely situated conductors induces a voltage on the second conductor. It was recently shown that the drag voltage must have a large mesoscopic component,

22. ''Mesoscopic fluctuations of the Coulomb drag,'' B.N. Narozhny and I.L. Aleiner, Phys. Rev. Lett. **84**, 5383–5386 (2000). (A)

Although the conductance and other observable characteristics for a given device may deviate from their sample-averaged value, there are symmetry relations these quantities must obey. The Onsager–Casimir symmetry relations for the sample-specific conductance are deduced in

23. ''Four-terminal phase-coherent conductance,'' M. Büttiker, Phys. Rev. Lett. **57**, 1761–1764 (1986). (I)

An extensive list of references to the papers on UCF in disordered conductors can be found in the published earlier Resource Letter QIMS-1,

24. ''Resource Letter QIMS-1: Quantum Interference in Macroscopic Samples,'' S. Das Sarma, T. Kawamura, and S. Washburn, Am. J. Phys. **63**, 683 (1995). (E)

IV. QUANTUM CHAOS AND MESOSCOPIC CONDUCTANCE FLUCTUATIONS

For the observation of UCF, propagation of electrons across the sample should be coherent, but not necessarily ballistic (i.e., electrons can encounter scatterers before reaching the boundaries of the sample). In fact, the experiments mentioned in Sec. III were performed with disordered samples, so that the electron mean free path was typically much smaller than the sample size. What happens with the conduction if the electron propagation is ballistic across the sample? It turns out that the universal nature of mesoscopic conductance fluctuations survives, as long as the electron motion within the sample is chaotic, and the sample is ''open,'' i.e., the conductance of contacts to the sample significantly exceeds e^2/h. The former condition is easily satisfied in samples without spatial symmetries; the results of violation of the latter condition will be discussed later.

Semiclassical theory of transport in open ballistic microstructures is reviewed in:

25. ''Quantum-chaotic scattering effects in semiconductor microstructures,'' H.U. Baranger, R.A. Jalabert, and A.D. Stone, Chaos **3**, 665–682 (1993). (E)

26. ''Transport Theory of Mesoscopic Systems: Application to Ballistic Transport,'' A.D. Stone, pp. 325–372 in Ref. 4. (I)

27. "Chaos in Ballistic Nanostructures. I. Theory," H.U. Baranger, in Ref. 7. (I)

Theory of mesoscopic fluctuations in open devices with an emphasis on quantum dots is also reviewed in Sec. IV of the article

28. "The Statistical Theory of Quantum Dots," Y. Alhassid, Rev. Mod. Phys. **72**, 896–968 (2000). (I)

The UCF, and even the statistical distribution of the values of the conductance in the case of quantum-chaotic motion of electrons, can be found within the phenomenological random matrix theory (RMT), originally developed for the description of spectra of complex nuclei. For a comprehensive review of RMT approach to quantum transport, see:

29. "Random-matrix theory of quantum transport," C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731–816 (1997). (I)

The use of RMT can be justified microscopically by using the supersymmetry method. The application of this method to the quantum-chaos problem and its relation to the RMT is discussed in a pedagogical way in the review

30. "Quantum Chaos: A Field Theory Approach," O. Agam, A.V. Andreev, and B.D. Simons, Chaos, Solitons, Fractals **8**, 1099–1129 (1997). (I)

The supersymmetry method and its applications to physical systems are also presented in the review

31. "Universalities: From Anderson Localization to Quantum Chaos," B.L. Altshuler and B.D. Simons, pp. 1–98 in Ref. 4. (A)

and in the book:

32. Supersymmetry in Disorder and Chaos, K.B. Efetov (Cambridge U.P., Cambridge, 1998). (A)

Experiments on conductance fluctuations in the quantum-chaotic regime of electron propagation are performed on quantum dots—small patches of two-dimensional electron gas prepared in semiconductor heterostructures. Because of high electron mobility in heterostructures, it is possible to reach a regime in which the electron mean free path exceeds the size of a quantum dot. In addition, the dots can be made irregularly shaped by a special design of the gates confining the electron gas. The experiments and their relation to the statistical theories of the conductance fluctuations are reviewed in:

33. "Conductance fluctuations and quantum chaotic scattering in semiconductor microstructures," C.M. Marcus, R.M. Westervelt, P.F. Hopkins, and A.C. Gossard, Chaos **3**, 643–653 (1993). (E)

34. "Chaos in Ballistic Nanostructures. II. Experiment," R.M. Westervelt, in Ref. 7. (E)

The above papers consider statistical fluctuations of the dc conductance. These fluctuations can be revealed by studying an ensemble of samples (an ensemble may be created by changing the sample's shape, or just by applying magnetic field to a sample). Temporal fluctuations—noise—also acquire new features on the mesoscopic scale. There is a very recent review on this subject,

35. "Shot noise in mesoscopic conductors," Ya.M. Blanter and M. Buttiker, Phys. Rep. **336**, 1–166 (2000). (I)

It contains up-to-date references to the relevant experimental and theoretical work.

Along with the conductance, other electronic characteristics of quantum-chaotic systems also display mesoscopic fluctuations, much like conductors with a relatively short mean free electron path. In addition to the examples of Sec. III (see Refs. 19–21), here I mention *adiabatic pumping* of electrons. In a pumping experiment, the potential confining the electrons changes cyclically in time, whereas the bias applied to the device is zero. In each cycle, a certain amount of charge is transferred through the device. The magnitude and even the sign of the pumping current depend on the interference pattern of the electron waves in the quantum-chaotic system, and therefore are sensitive to the application of a weak magnetic field. Experimental results and references to the theoretical papers devoted to the adiabatic pumping are presented in

36. "An Adiabatic Quantum Electron Pump," M. Switkes, C.M. Marcus, K. Campman, and A.C. Gossard, Science **283**, 1908–1908 (1999). (I)

V. ELECTRON TRANSPORT IN QUANTUM POINT CONTACTS

Soon after the discovery of UCF, the wave properties of an electron in a solid were demonstrated spectacularly in experiments with quantum point contacts. In the first successful experiments, these devices were based on gated GaAs heterostructures. The gates allow one to control the width of a constriction formed in a two-dimensional electron gas. Such a constriction works essentially as a waveguide for the electrons (see Sec. I). By widening the constriction, it is possible to increase the number of propagating modes one by one. Each additional propagating mode contributes a unit quantum, $2e^2/h$, to the conductance of the constriction (this follows from the *Landauer formula* relating the transmission coefficient and conductance; see, e.g., Refs. 2, 8, 9, and 23 above).

Conductance of quantum point contacts formed in GaAs heterostructures by means of gate depletion is described in popular articles,

37. "Ballistic Electron Transport through a Narrow Channel is Quantized," A. Khurana, Phys. Today **41**, 21–23 (1988). (E)

38. "Quantum Point Contacts," H. van Houten and C. Beenakker, Phys. Today **49** (7), 22–27 (1996). (E)

The original experiments were reported in

39. "Quantized conductance of point contacts in a two-dimensional electron gas," B.J. van Wees, H. van Houten, C.W.J. Beenakker, J.G. Williamson, L.P. Kouwenhoven, D. van der Marel, and C.T. Foxon, Phys. Rev. Lett. **60**, 848–850 (1988). (E)

40. "One-dimensional transport and the quantization of the ballistic resistance," D.A. Wharam, T.J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J.E.F. Frost, D.G. Hasko, D.C. Peacock, D.A. Ritchie, and G.A.C. Jones, J. Phys. C **21**, 209–214 (1988). (E)

The relation between the quantization of conductance and the geometry of the junction (which must be smooth on the scale defined by the Fermi wavelength of the propagating electrons for the quantization to occur) was established in the theoretical paper

41. "Reflectionless quantum transport and fundamental ballistic-resistance steps in microscopic constrictions," L.I. Glazman, G.B. Lesovik, D.E. Khmel'nitskii, and R.I. Shekhter, JETP Lett. **48**, 238–241 (1988) [Pis'ma Zh. Eksp. Teor. Fiz. **48**, 218–220 (1988)]. (E)

A detailed account of the conductance quantization phenomenon can be found in Ref. 2. A description of the main experiments and pedagogical presentation of theory is given in:

42. ‘‘Quantization of the Transport,’’ B. Kramer, pp. 126–132 in *Quantum Transport and Dissipation*, edited by T. Dittrich, P. Hänggi, G.-L. Ingold, B. Kramer, G. Schön, and W. Zwerger (Wiley–VCH, Weinheim, 1998). (E)

In addition to the well-understood integer conductance steps, a number of research groups report observations of a step-like feature at approximately $0.7\times 2e^2/h$; see, e.g.,

43. ‘‘Spin properties of low-density one-dimensional wires,’’ K.J. Thomas, J.T. Nicholls, M. Pepper, W.R. Tribe, M.Y. Simmons, and D.A. Ritchie, Phys. Rev. B **61**, 13365–13368 (2000). (I)

There is no commonly accepted explanation of this feature.

There also are reports of observation of the conductance steps in metallic junctions. The energy separation of transverse modes in a point contact of atomic dimensions is so large that the conductance steps are visible at room temperature. Apparently, the effect is sufficiently robust for reproducing it in a classroom setting, see

44. ‘‘Do-It-Yourself Quantum Mechanics,’’ Phys. Today **49**, 9 (1996). (E)

There is still no full agreement between various groups on the results of experiments with metallic quantum point contacts, and on the interpretation of the observations. As an example of somewhat different perspectives on the phenomenon, see

45. ‘‘Quantum Point Contacts Between Metals,’’ J.M. van Ruitenbeek, pp. 549–580 in Ref. 6. (E)

46. ‘‘Conductance Quantization in Metallic Nanowires,’’ N. Garcia, J.L. Kosta-Krämer, A. Gil, M.I. Marqués, and A. Corriea, pp. 581–616 in Ref. 6. (E)

Current flowing through a point contact does not produce any shot noise if the contact is in a state with quantized conductance; noise appears only if the contact is tuned to the step connecting two adjacent plateaus of the conductance. (Shot noise results from the discreteness of the electron charge.) This striking difference of the quantum point contact from a ‘‘classical’’ conductor was predicted in

47. ‘‘Excess quantum noise in 2D ballistic point contacts,’’ G.B. Lesovik, JETP Lett. **49**, 592–594 (1989) [Pis'ma Zh. Eksp. Teor. Fiz. **49**, 513–515 (1989)]. (E)

and experimentally observed in

48. ‘‘Temporal correlation of electrons: Suppression of shot noise in a ballistic quantum point contact,’’ M. Reznikov, M. Heiblum, H Shtrikman, and D. Mahalu, Phys. Rev. Lett. **75**, 3340–3343 (1995). (E)

and in

49. ‘‘Experimental test of the quantum shot noise reduction theory,’’ A. Kumar, L. Saminadayar, D.C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **76**, 2778–2781 (1996). (E)

Furthermore, the magnitude of noise depends on the value of charge of the quasiparticles responsible for the conduction. Theory predicts that in the conditions of the fractional quantum Hall effect, the elementary charges are certain fractions of the electron charge. This prediction was checked successfully in experiments with a point contact formed in a two-dimensional electron gas placed in a strong magnetic field:

50. ‘‘Direct observation of a fractional charge,’’ R. De-Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahalu, Nature (London) **389**, 162–164 (1997). (E)

51. ‘‘Observation of the $e/3$ fractionally charged Laughlin quasiparticle,’’ L. Saminadayar, D.C. Glattli. Y. Jin, and B. Etienne, Phys. Rev. Lett. **79**, 2526–2529 (1997). (E)

A review of these experiments is published in

52. ‘‘Quantum shot noise,’’ M. Reznikov, R. de Picciotto, M. Heiblum, D.C. Glattli, A. Kumar, and L. Saminadayar, Superlattices and Microstruct. **23**, 901–915 (1998). (E)

For a recent review on shot noise in mesoscopic systems, see Ref. 34 in Sec. IV.

VI. INTERACTING ELECTRONS IN NANOWIRES

Electron transport in conductors is conventionally described in terms of Landau quasiparticles forming the Fermi liquid. Unlike the underlying ‘‘real’’ electrons, the quasiparticles interact only weakly with each other, and their individual quantum states are characterized by long lifetimes. The Fermi liquid theory, which works remarkably well in higher dimensions, breaks down in one-dimensional conductors. Electron–electron interaction there destroys quasiparticles, and another language, *Luttinger liquid* theory, is needed to describe the observable properties of one-dimensional conductors. It was predicted that even a single scatterer in a Luttinger liquid brings the conductance down to zero; see

53. ‘‘Transport in a One-channel Luttinger Liquid,’’ C.L. Kane and M.P.A. Fisher, Phys. Rev. Lett. **68**, 1220 (1992). (I)

Theory of transport through a single impurity is reviewed, from several different perspectives, in:

54. ‘‘Transport in a One-dimensional Luttinger Liquid,’’ M.P.A. Fisher and L.I. Glazman, pp. 331–373 in Ref. 6. (I)

A more formal review, concentrating on the techniques used in the theory of Luttinger liquid, is given in:

55. ‘‘Fermi Liquids and Non-Fermi Liquids,’’ H.J. Schulz, pp. 533–603 in Ref. 4. (I)

Note that the conductance of an ideal Luttinger liquid without scatterers is independent of the interaction strength, and coincides with the conductance of free electrons in a one-dimensional system, see, e.g.,

56. ‘‘Landauer conductance of Luttinger liquids with leads,’’ D.L. Maslov, M. Stone, Phys. Rev. B **52**, 5539–5542 (1995). (I)

An excellent introduction to the general Luttinger liquid theory can be found in:

57. ‘‘Luttinger liquid theory' of one-dimensional quantum fluids. I. Properties of the Luttinger model and their extension to the general 1D interacting spinless Fermi gas,’’ F.D.M. Haldane, J. Phys. C **14**, 2585 (1981). (I)

Theoretical predictions made for the conduction of a Luttinger liquid containing scatterers were checked in experiments with carbon nanotubes:

58. ‘‘Luttinger-liquid behaviour in carbon nanotubes,’’ M. Bockrath, D.H. Cobden, Jia Lu, A.G. Rinzler, R.E. Smalley, L. Balents, and P.L. McEuen, Nature (London) **397**, 598–601 (1999). (I)

59. ‘‘Carbon nanotube intramolecular junctions,’’ Z. Yao, H.W.Ch. Postma, L. Balents, and C. Dekker, Nature (London) **402**, 273–276 (1999). (I)

The relevant experimental material is presented also in two recent popular articles:

60. ‘‘Single-wall carbon nanotubes,’’ P.L. McEuen, Phys. World **13**, 31–36 (2000). (E)

61. ‘‘Carbon nanotubes as molecular quantum wires,’’ C. Dekker, Phys. Today **52**, 22–28 (May 1999). (E)

Edge states formed at the boundaries of the two-dimensional electron gas driven into the regime of the quantum Hall effect by a strong magnetic field provide another important example of a one-dimensional system. A simplified picture for the edge states in the *integer* quantum Hall effect regime is given in the review article:

62. ''Quantum Electrical Transport in Samples of Limited Dimensions,'' D.F. Holcomb, Am. J. Phys. **67**, 279–297 (1998). (E)

In the *fractional* quantum Hall effect regime, the edge excitations are believed to be described by a one-dimensional chiral Luttinger liquid; see Sec. 3 of the review:

63. ''Topological Orders and Edge Excitations in FQH States,'' X.-G. Wen, Adv. Phys. **44**, 405–474 (1995). (I)

At this moment, there is no full understanding of the data obtained in experiments that were aimed at checking the theoretical predictions; see the latest experimental paper,

64. ''Plateau Behavior in the Chiral Luttinger Liquid Exponent,'' A.M. Chang, M.K. Wu, C.C. Chi, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **86**, 143 (2001). (I)

and references therein.

VII. COULOMB BLOCKADE AND CONDUCTANCE OF QUANTUM DOTS

An electron that tunnels between massive leads through a small conductor charges it. If the contacts to the small conductor are weak (their conductance is significantly less than e^2/h), then the charge of the conductor is well defined. As was mentioned in Sec. I, at low temperatures T the electrostatic energy E_C associated with the charge of one extra electron may prevent the thermally activated hopping of an electron through the small conductor, leading to the *Coulomb blockade* of tunneling. The role of a small conductor may be played by a metallic island (''single-electron box'') prepared by e-beam lithography, a small metallic grain, a semiconductor quantum dot, or even by a single large molecule. The main difference between the metallic single-electron boxes and semiconductor quantum dots (or smaller metallic grains with sizes in the 30- to 50-nm range) is in the value of the typical energy spacing δE between closest individual quasiparticle states. In ''single-electron boxes'' δE is fairly small, and therefore individual levels cannot be resolved even at sub-1-K temperatures. The main parameter defining the observable properties of a single-electron box is the dimensionless ratio E_C/k_BT; Coulomb blockade develops at $E_C/k_BT \gg 1$. In a semiconductor quantum dot, the level spacing δE easily may exceed thermal energy, $\delta E \gtrsim k_BT$; in this case, properties of individual quasiparticle states are resolvable in electron transport measurements.

The number of papers devoted to the experimental and theoretical aspects of the Coulomb blockade phenomenon is very substantial and continues to grow rapidly. Luckily there were several comprehensive reviews and popular articles published quite recently; I focus on these two categories of publications in this section.

A popular and instructive description of the Coulomb blockade in single-electron boxes, along with the discussion of their possible device applications, can be found in:

65. ''Single-electron Transistors,'' Michel Devoret and Christian Glattli, Phys. World **11**, 29–33 (1998). (E)

A presentation of the Coulomb blockade physics geared toward the use in an undergraduate course of physics was published in the ''New Problems'' section of this journal,

66. ''Coulomb Blockade and Electron in a Mesoscopic Box,'' M. Tinkham, Am. J. Phys. **64**, 343–347 (1996). (E)

A number of relevant tutorials for the Coulomb blockade in single-electron boxes based on lectures presented at the NATO Advanced Study Institute on Single Charge Tunneling (Les Houches, France, 1991) is published as a book; see Ref. 3 above. Specifically, basic concepts of the charge transfer through small metallic islands are reviewed in

67. ''Transferring Electrons One by One,'' D. Esteve, pp. 109–138 in Ref. 3. (E)

The process of charging of tiny metallic box is inevitably associated with the problem of charge spreading in the course of tunneling through a small junction (the ''electromagnetic-environment'' effect). This effect is reviewed in

68. ''Charge Tunneling Rates in Ultrasmall Junctions,'' G.-L. Ingold and Yu.V. Nazarov, pp. 21–108 in Ref. 3. (I)

The main manifestation of the effect of ''electromagnetic environment'' is the suppression of the low-bias conductance of a small tunnel junction. This manifestation is identical to the interaction-induced zero-bias anomaly in tunneling, known for a long time in a somewhat different context; see

69. ''Electron–Electron Interaction in Disordered Conductors,'' B.L. Altshuler and A.G. Aronov, in *Electron–Electron Interactions in Disordered Systems*, edited by A.L. Efros and M. Pollak (Elsevier, Amsterdam, 1985), pp. 1–75. (A)

In fact, the physics of the ''electromagnetic-environment'' effect is the same as that of the zero-bias anomaly. Apparently, this fact became fully appreciated only recently; see

70. ''Electrodynamic Dip in the Local Density of States of a Metallic Wire,'' F. Pierre, H. Pothier, P. Joyez, N.O. Birge, D. Esteve, and M.H. Devoret, Phys. Rev. Lett. **86**, 1590–1593 (2001). (I)

Elementary processes of electron transfer through a blockaded metallic island or a quantum dot are reviewed in

71. ''Macroscopic Quantum Tunneling of Charge and Co-Tunneling,'' D.V. Averin and Yu.V. Nazarov, pp. 217–246, in Ref. 3. (I)

A simple description of these processes, with the statistical properties and the temperature intervals of relevance for each of them, is presented in

72. ''Mesoscopic Fluctuations of Co-tunneling and Kondo Effect in Quantum Dots,'' L.I. Glazman, in *Quantum Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics*, edited by I.O. Kulik and R. Ellialtioglu (Proceedings of NATO Advanced Study Institute, Kluwer Academic, Dordrecht, 2000), pp. 105–128. (E)

Experiments on electron transport through quantum dots are well described in the popular articles,

73. ''Artificial atoms'' M. Kastner, Phys. Today **46**, 24–31 January (1993). (E)

74. ''Quantum dots,'' L.P. Kouwenhoven and C.M. Marcus, Phys. World **11**, 35–38 (1998). (E)

Large level spacing δE in quantum dots enables one to study electron shell effects and the statistical properties of individual quantum states in a dot; see Ref. 74. It also makes it possible to observe an elegant many-body phenomenon—the

Kondo effect. This effect occurs only if a quantum dot carries a nonzero spin. Quantum coherence between the dot's spin and spins of the conduction electrons in the leads sets in at low temperatures, and results in an anomalously large conductance through the dot (lifting of the Coulomb blockade). A popular description of the manifestations of the Kondo effect and references to the original papers can be found in

75. ''Revival of the Kondo Effect,'' L. Kouwenhoven and L. Glazman, Phys. World **14**, 33–38 (2001). (E)

A broad description of the properties of quantum dots can be found in the review

76. ''Electron Transport in Quantum Dots,'' L.P. Kouwenhoven, C.M. Marcus, P.L. McEuen, S. Tarucha, R.M. Westervelt, and N.S. Wingreen, pp. 105–214 in Ref. 6. (I)

Further experimental data on the statistical properties of electron transport are presented in review articles.

77. ''Quantum Chaos in Open vs Closed Quantum Dots: Signatures of Interacting Particles,'' C.M. Marcus, S.R. Patel, A.G. Huibers, S.M. Cronenwett, M. Switkes, I.H. Chan, R.M. Clarke, J.A. Folk, and S.F. Godijn, Chaos, Solitons Fractals **8**, 1261–1279 (1997). (I)
78. ''Quantum Chaos in Ga/Al$_x$Ga$_{1-x}$As Microstructures,'' A.M. Chang, Chaos, Solitons Fractals **8**, 1281–1297 (1997). (I)

A review of the statistical theory of quantum dots in the Coulomb blockade regime, with references to the relevant experimental material, can be found in Ref. 28, Secs. V–VII. Theory of mesoscopic fluctuations in the Coulomb blockade regime, Kondo effect, and crossover to the ''open-dot'' regime are described in detail in the review:

79. ''Quantum Effects in Coulomb Blockade,'' I.L. Aleiner, P.W. Brouwer, and L.I. Glazman, arXiv.org/abs/cond-mat/0103008 (2001). (A)

Most of the theoretical papers devoted to the Kondo effect in quantum dots start with a slight extension of the Anderson impurity model,

80. ''Localized Magnetic States in Metals,'' P.W. Anderson, Phys. Rev. **124**, 41–53 (1961). (I)

For the application of this model to the low-temperature tunneling effects, see

81. ''Resonant Kondo transparency of a barrier with quasilocal impurity states,'' L.I. Glazman and M.E. Raikh, JETP Lett. **47**, 452–455 (1988) [Pis'ma Zh. Eksp. Teor. Fiz. **47**, 378–380 (1988)]. (I)
82. ''On-site Coulomb repulsion and resonant tunneling,'' T.K. Ng and P.A. Lee, Phys. Rev. Lett. **61**, 1768–1771 (1988). (I)

However, a complicated electron level structure of the dot is important in a number of effects related to the Kondo physics. Some modifications of the Kondo effect, specific for the quantum dots, are described in the review article

83. ''Magnetic Field-induced Kondo Effects in Coulomb Blockade Systems,'' M. Pustilnik, L.I. Glazman, D.H. Cobden, and L.P. Kouwenhoven, in **Lecture Notes in Physics**, edited by R. Haug and H. Schoeller, **579**, 3–24 (2001). (I)

which combines the experimental results with their theoretical explanations.

Transport through very small metallic particles in many respects is similar to the transport through quantum dots, despite very different experimental techniques used to study it; see

84. ''Tunneling through Metallic Quantum Dots,'' M. Tinkham, D. Davidovic, D.C. Ralph, and C.T. Black, J. Low Temp. Phys. **118**, 271–286 (2000) [Special Issue: Proceedings of the International Conference on the Electron Transport in Mesoscopic Systems, ETMS-99]. (I)
85. ''Spectroscopy of Discrete Energy Levels in Ultrasmall Metallic Grains,'' J. von Delft and D.C. Ralph, Phys. Rep. **345**, 61 (2001). (I)

VIII. MESOSCOPIC SUPERCONDUCTIVITY

There are a number of directions in the ''traditional'' superconductivity that have natural links to mesoscopic physics. It is sufficient to mention the Josephson effect, Andreev reflection, and the proximity effect to see how difficult it is to draw a boundary between the two fields. An introduction to many directions in mesoscopic superconductivity can be found in Chap. 7 of the book:

86. Introduction to Superconductivity, M. Tinkham (McGraw–Hill, New York, 1996), 2nd ed. (I)

In this section, references are given only to several reviews on the following subjects: the number parity effect in superconducting islands, superconductivity in ultra-small grains, and the anomalous proximity effect.

In a superconductor, electrons tend to be bound in pairs. Therefore, a grain with an odd number of electrons has a single unpaired electron, which increases the ground state energy of the grain by the value of the superconducting gap Δ (we ignore the level spacing δE here). This yields observable modifications of the thermodynamics of a single grain, as well as of the electron transport through it. Theory of the odd–even effect and references to the relevant experimental papers can be found in a short review article,

87. ''Effects of Charge Parity in Tunneling through a Superconducting Grain,'' K.A. Matveev, L.I. Glazman, and R.I. Shekhter, Mod. Phys. Lett. B **8**, 1007–1026 (1994). (I)

As long as $\Delta \gg \delta E$, the finite size of the grain does not affect the superconducting pairing in it. In ultrasmall particles, however, the level spacing becomes large enough, $\delta E \sim \Delta$, to interfere with the pairing mechanism (for the estimates of the relevant energy scales, see Ref. 56). The physics (experiment and theory) of such small superconducting grains was reviewed recently in Ref. 84 of Sec. VII.

The proximity effect and Andreev reflection in mesoscopic junctions of a superconductor with a normal metal or a semiconductor is reviewed in

88. ''The Proximity Effect in Mesoscopic Diffusive Conductors,'' D. Estéve, H. Pothier, S. Guéron, N.O. Birge, and M.H. Devoret, pp. 375–406 in Ref. 6. (I)
89. ''Andreev Reflection and Proximity Effect,'' B. Pannetier and H. Cortois, J. Low Temp. Phys. **118**, 599–615 (2000) [Special Issue: Proceedings of the International Conference on the Electron Transport in Mesoscopic Systems, ETMS-99]. (I)
90. ''The Superconducting Proximity Effect in Semiconductor-Superconductor Systems: Ballistic Transport, Low Dimensionality and Sample Specific Properties,'' B.J. van Wees and H. Takayanagi, pp. 469–502 in Ref. 6. (I)

Note that Ref. 89 contains some material on the properties of junctions between a superconductor and a conducting ferromagnet—an area of current active research.

IX. MESOSCOPIC PHYSICS AND NANOELECTRONIC DEVICES

The Coulomb blockade phenomenon is utilized in the single-electron transistor (SET), which can be used as a very sensitive electrometer; see Ref. 65. Recently substantial progress was made in shortening the read-out time for such an electrometer; see

91. ''The Radio-frequency Single-electron Transistor (RF-SET): A Fast and Ultra-Sensitive Electrometer,'' R.J. Schoelkopf, P. Wahlgren, A.A. Kozhevnikov, P. Delsing, and D.E. Prober, Science **280**, 1238–1242 (1998). (I)

Perspectives for the use of single-electron devices in highly integrated circuits are analyzed in the article

92. ''Overview of nanoelectronic devices,'' D. Goldhaber-Gordon, M.S. Montemerlo, J.C. Love, G.J. Opiteck, and J.C. Ellenbogen, Proc. IEEE **85**, 521–540 (1997). (E)

This article has an extensive bibliography.

Currently, device-oriented research in mesoscopic physics is fueled by the idea of quantum computing. Introduction to quantum computing for physicists can be found in the notes of N.D. Mermin (Cornell University), which can be downloaded from the Web site,

93. http://www.lassp.cornell.edu/ cew2/QuantComp.pdf. (E)

The idea of quantum computing transformed the seemingly esoteric question of the phase-coherent evolution of a physical system into a practically important issue about the feasibility of realization of a q-bit in a solid-state device (q-bit is an element of a quantum computer). Coherent time evolution of a superconducting Coulomb blockade device has been demonstrated by now. The description of this experiment and references to other ideas for q-bits can be found in the review article

94. ''Quantum-state Control with a Single-Cooper Pair Box,'' Y. Nakamura and J.S. Tsai, J. Low Temp. Phys. **118**, 765–780 (2000) [Special Issue: Proceedings of the International Conference on the Electron Transport in Mesoscopic Systems, ETMS-99]. (I)

The section on quantum computing of this special issue of the Journal of Low Temperature Physics also contains a number of papers that reflect the state of theory of mesoscopic realizations of q-bits.

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METAPHORS AND QUANTUM PHYSICS

Of the sciences today, quantum physics alone seems to have found its way back to an equitable relationship with metaphors, those fundamental tools of the imagination. The other sciences are occasionally so bound by rational analysis, or so wary of metaphor, that they recognize and denounce anthropomorphism as a kind of intellectual cancer, instead of employing it as a tool of comparative inquiry, which is perhaps the only way the mind works, that parallelism we finally call narrative.

Barry Lopez, *Artic Dreams* (Scribners, New York, NY, 1986), p. 250.

Submitted by Gary E. Bowman.