QCD Matter at Very Low Density

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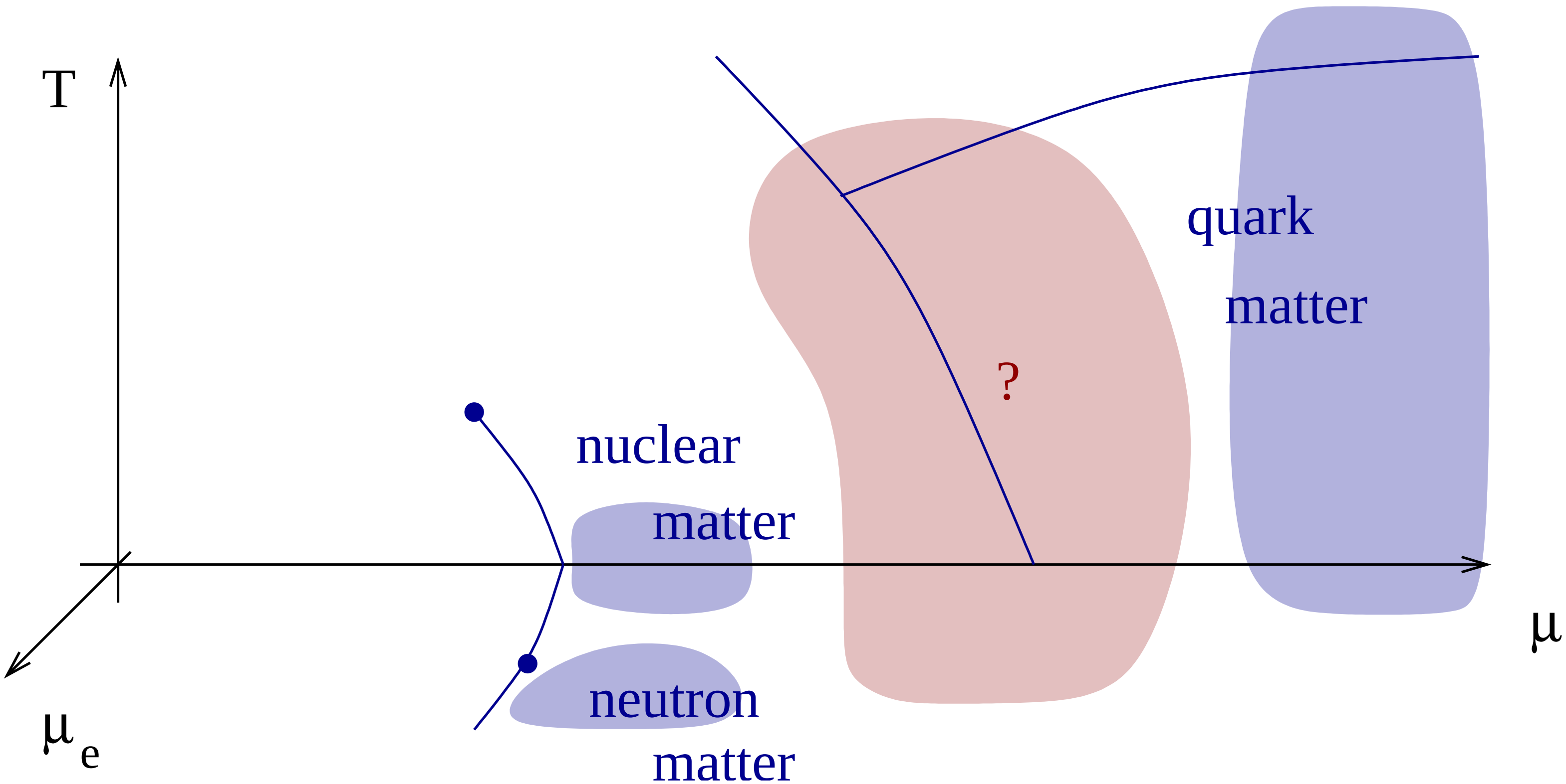
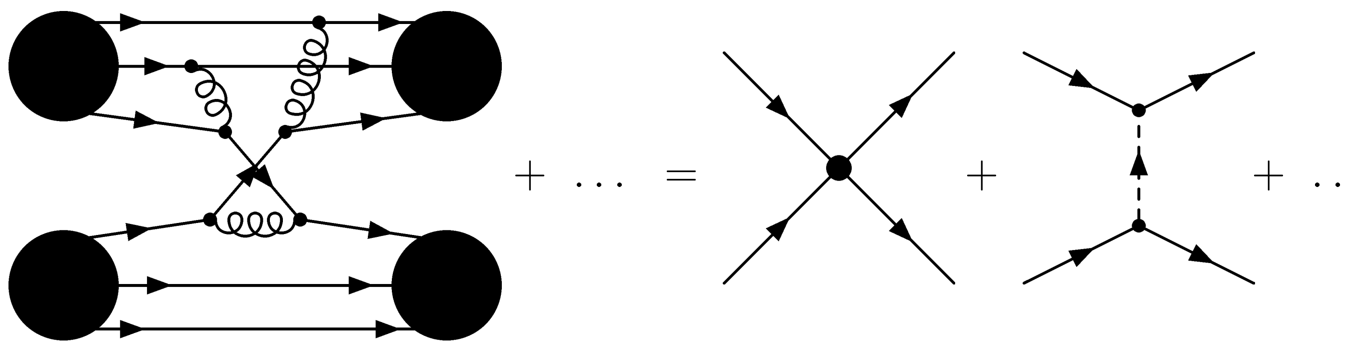
Schematic Phase Diagram of Dense Matter



Nuclear Effective Field Theory

Low Energy Nucleons:
- Nucleons are point particles
- Interactions are local
- Long range part: pions



Advantages:
- Systematically improvable
- Symmetries manifest (Chiral, gauge, ...)
- Connection to lattice QCD

Effective Field Theory

Effective field theory for pointlike, non-relativistic neutrons

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \dots$$

Simplifications: neutrons only, no pions (very low energy)

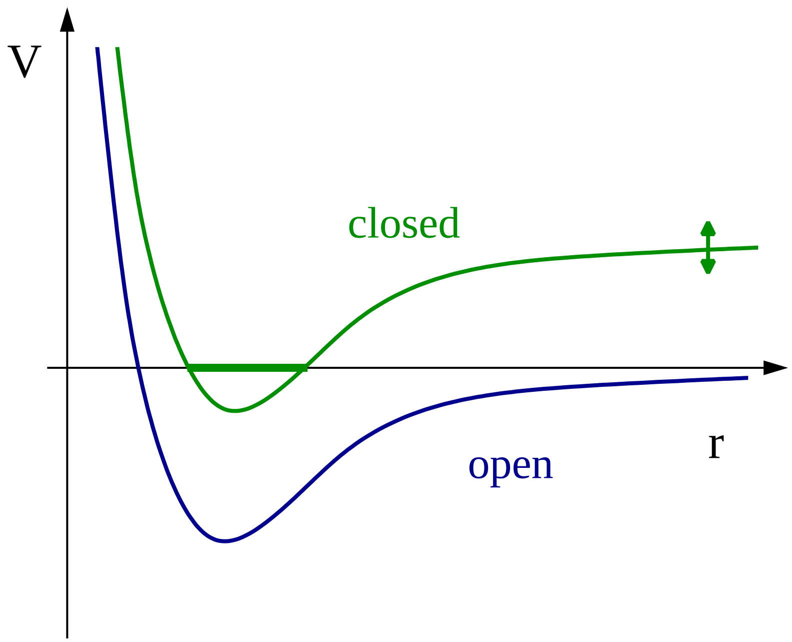
Match to effective range expansion

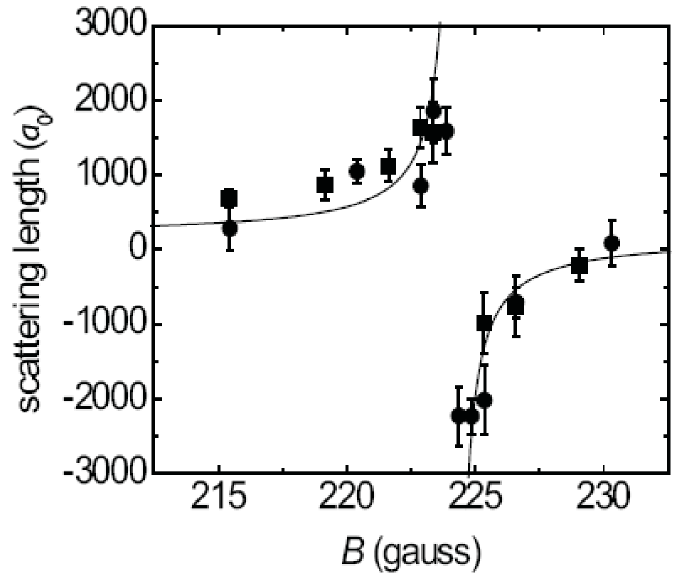
$$C_0 = \frac{4\pi a}{M} \qquad a = -18\,\text{fm}$$

$$C_2 = \frac{4\pi a^2}{M} \frac{r}{2} \qquad r = 2.8\,\text{fm}$$

Designer Fluids

Atomic gas with two spin states: "↑" and "↓"





Feshbach resonance

$$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

"Unitarity" limit $a \to \infty$

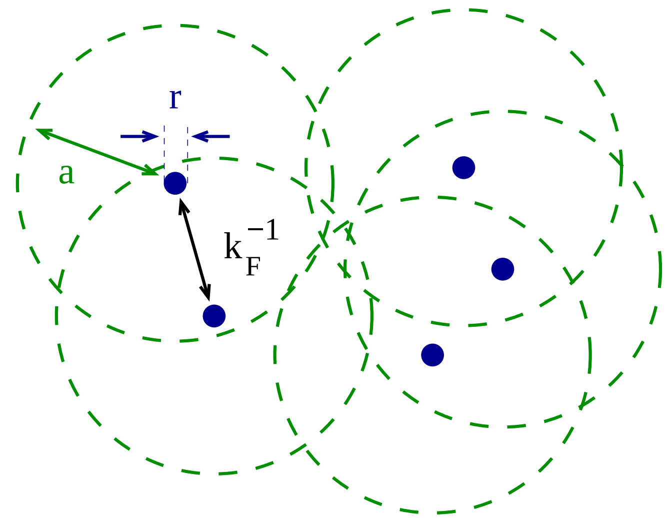
$$\sigma = \frac{4\pi}{k^2}$$

Universality

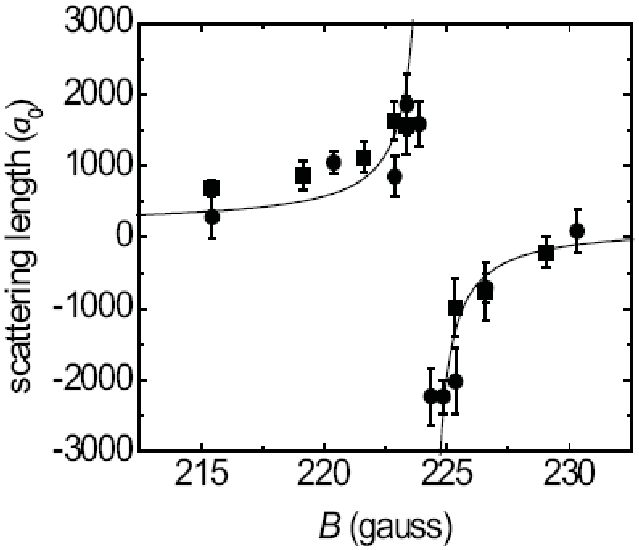
What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

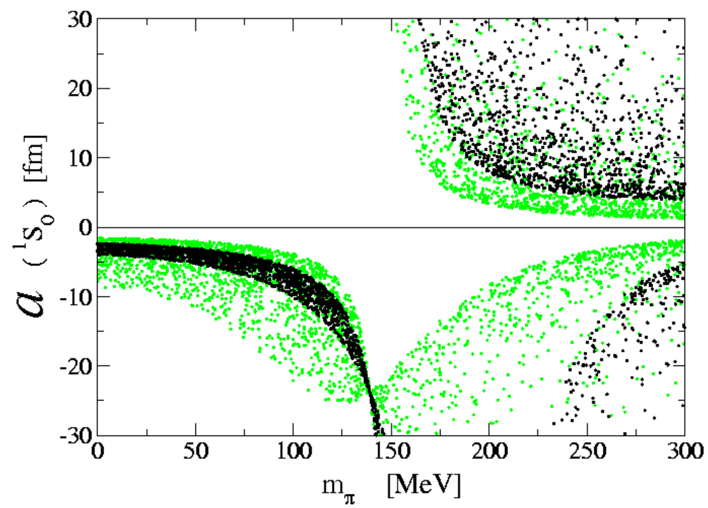
strongly correlated: $a\rho^{1/3} \gg 1$



Feshbach Resonance in ^{6}Li



Neutron Matter



Why are these systems interesting?

System is intrinsically quantum mechanical

cross section saturates unitarity bound

Scale (and conformally) invariant at unitarity

$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0, \qquad \text{OPE, power laws, } \ldots$$

System is strongly coupled but dilute

$$(k_F a) \to \infty \qquad (k_F r) \to 0$$

Strong hydrodynamic elliptic flow observed experimentally

I. Density Functional Theory

Effective Lagrangian

Low energy ($\omega < \Delta \sim E_F$) effective lagrangian

$$\mathcal{L} = c_0 m^{3/2} X^{5/2} + c_1 m^{1/2} \frac{(\vec{\nabla} X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}} \left[\left(\nabla^2 \varphi\right)^2 - 9m \nabla^2 A_0 \right] \sqrt{X}$$

$$X = \mu - A_0 - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}$$

variables

φ: phase $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

μ: chemical potential

A_0: gauge potential

constrained by

$U(1)$ invariance

Galilean invariance

Scale invariance

Conformal invariance

Greiter et al. (1989); Son, Wingate (2005)

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Density Functional

Construct energy functional $\mathcal{E}[n(x)] = \mu n(x) - P[\mu - A_0(x)]$

$$\mathcal{E}(x) = n(x)A_0(x) + \frac{3 \cdot 2^{2/3}}{5^{5/3} m c_0^{2/3}} n(x)^{5/3}$$

$$- \frac{4}{45} \frac{2c_1 + 9c_2}{m c_0} \frac{\left(\nabla n(x)\right)^2}{n(x)} - \frac{12}{5} \frac{c_2}{m c_0} \nabla^2 n(x) + \ldots$$

Non-perturbative physics in $c_0, c_1, c_2, \ldots$

Use epsilon ($\epsilon = d - 4$) expansion

Upper and lower critical dimension

Zero energy bound state for arbitrary d

$$\psi''(r) + \frac{d-1}{r}\psi'(r) = 0 \qquad (r > r_0)$$

d=2: Arbitrarily weak attractive potential has a bound state

$$\xi(d=2) = 1$$

d=4: Bound state wave function $\psi \sim 1/r^{d-2}$. Pairs do not overlap

$$\xi(d=4) = 0$$

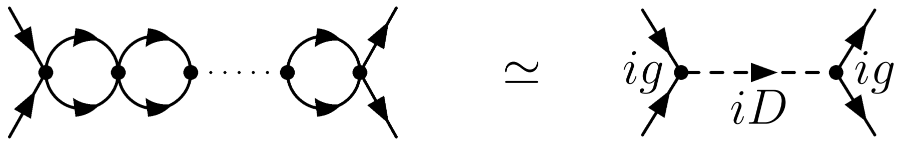
Conclude $\xi(d=3) \sim 1/2$?

Try expansion around $d = 4$ or $d = 2$?

Nussinov & Nussinov (2004)

Epsilon Expansion

EFT version: Compute scattering amplitude ($d = 4 - \epsilon$)



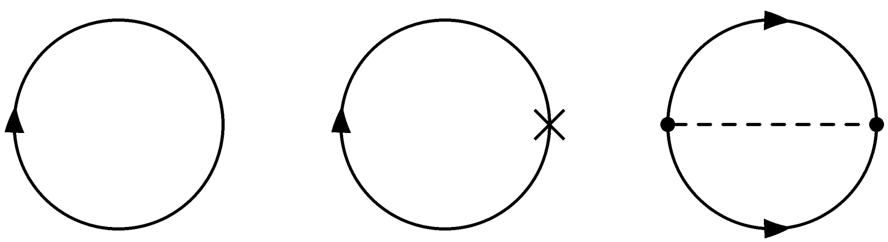
$$T = \frac{1}{\Gamma\left(1 - \frac{d}{2}\right)} \left(\frac{m}{4\pi}\right)^{-d/2} \left(-p_0 + \frac{\epsilon_p}{2}\right)^{1-d/2} \simeq \frac{8\pi^2\epsilon}{m^2} \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

$$g^2 \equiv \frac{8\pi^2\epsilon}{m^2} \qquad D(p_0, p) = \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

Weakly interacting bosons and fermions

Matching Calculations

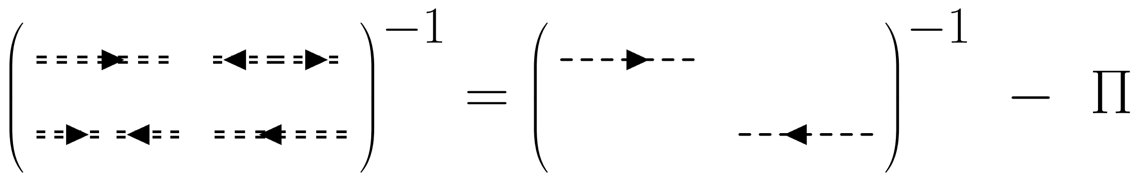
Effective potential

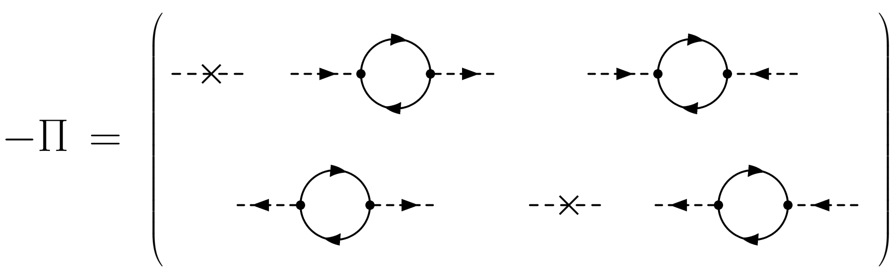


$O(1)$ $O(1)$ $O(\epsilon)$

$$P = \#(2m)^{d/2}\mu^{d/2+1}$$

Phonon Propagator

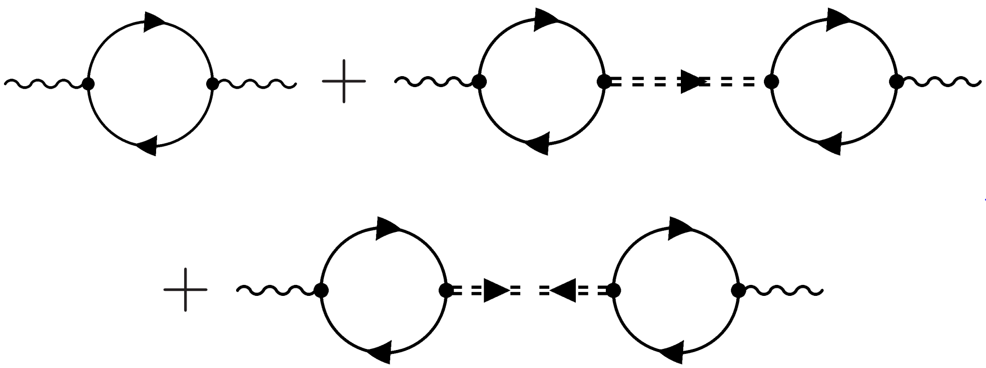




$$\omega = c_s p \left\{ 1 + \# \left(\frac{p^2}{m\mu} \right) + \dots \right\}$$

Matching (continued)

Static susceptibility $\chi(q) = \int d^3x\, e^{iqx}\, \langle \psi^\dagger\psi(x)\psi^\dagger\psi(0)\rangle$



$$\chi(q) = \chi(0)\left\{1 - \#\left(\frac{q^2}{m\mu}\right) + \ldots\right\}$$

Nishida, Son (2007)

Rupak, Schaefer (2008)

Density Functional

Unitarity Limit

$$\mathcal{E}(x) = n(x)A_0(x) + 1.364\,\frac{n(x)^{5/3}}{m} + 0.032\,\frac{(\nabla n(x))^2}{mn(x)} + O(\nabla^4 n)$$

$$E_{trap} = \frac{\sqrt{0.475}}{4}\omega(3N)^{4/3}\left(1 + \frac{2.4}{(3N)^{2/3}} + \ldots\right)$$

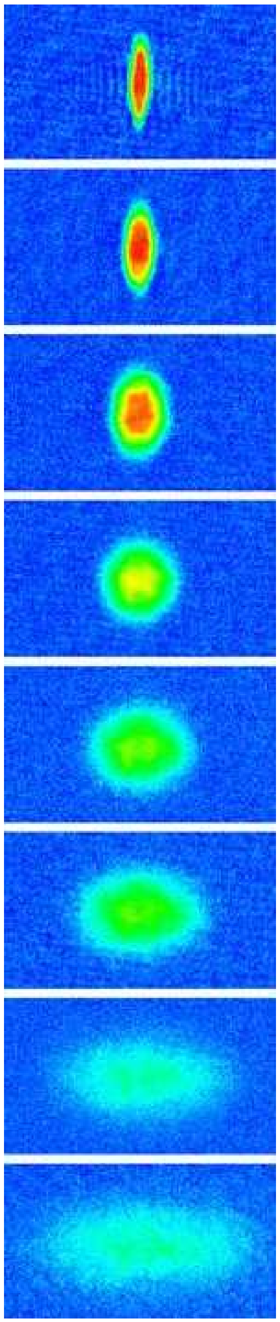
Free Fermions

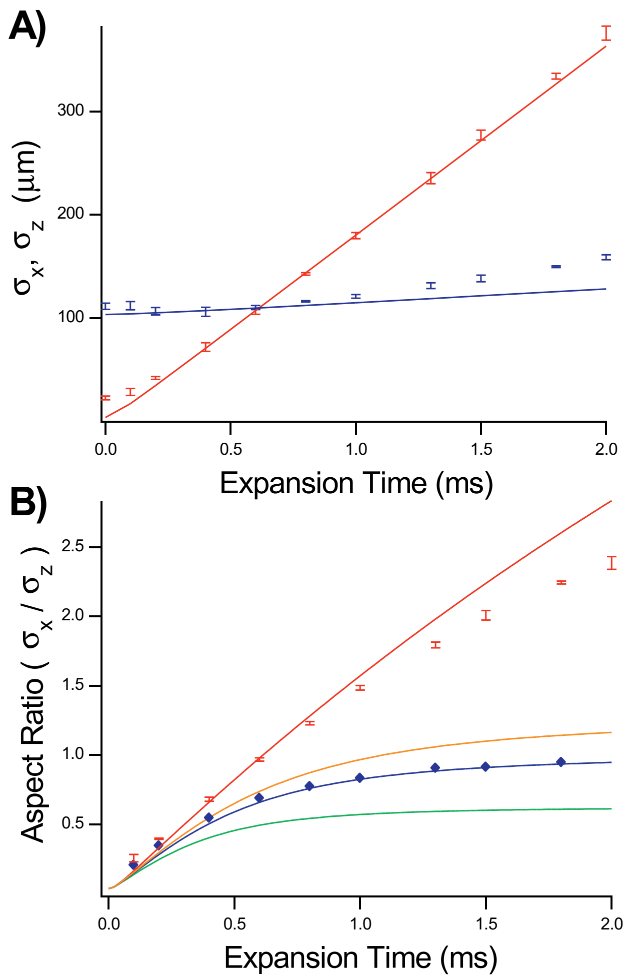
$$\mathcal{E}(x) = n(x)A_0(x) + 2.871\,\frac{n(x)^{5/3}}{m} + 0.014\,\frac{(\nabla n(x))^2}{mn(x)} + 0.167\,\frac{\nabla^2 n(x)}{m}$$

$$E_{trap} = \frac{1}{4}\omega(3N)^{4/3}\left(1 + \frac{0.5}{(3N)^{2/3}} + \ldots\right)$$

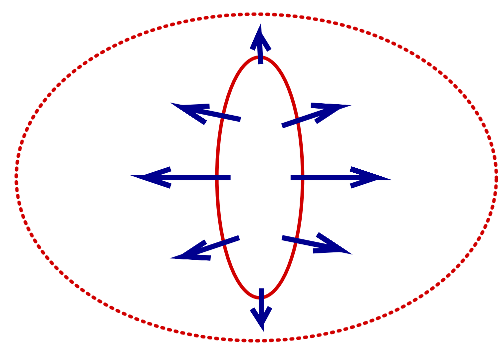
II. Transport Properties

Elliptic Flow



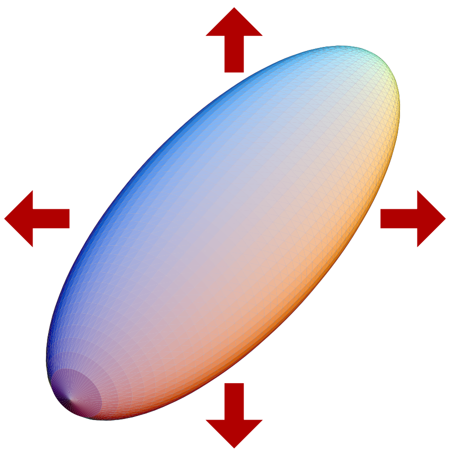


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

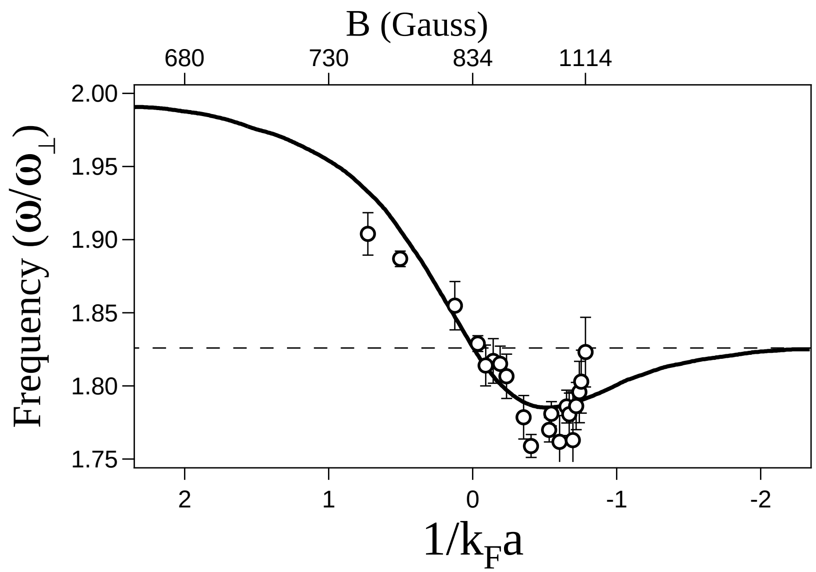


Collective Modes

Radial breathing mode



Kinast et al. (2005)



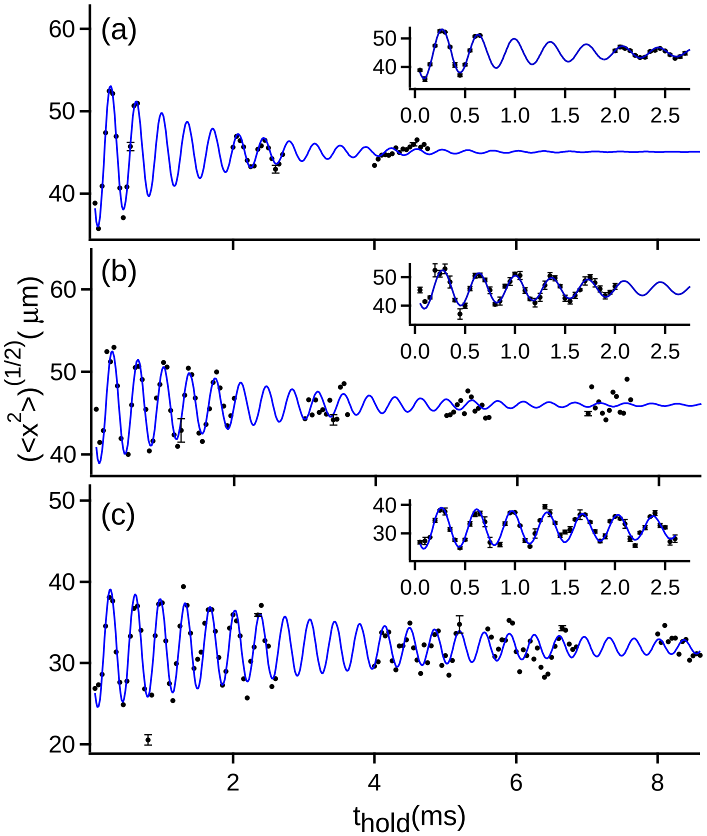
Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

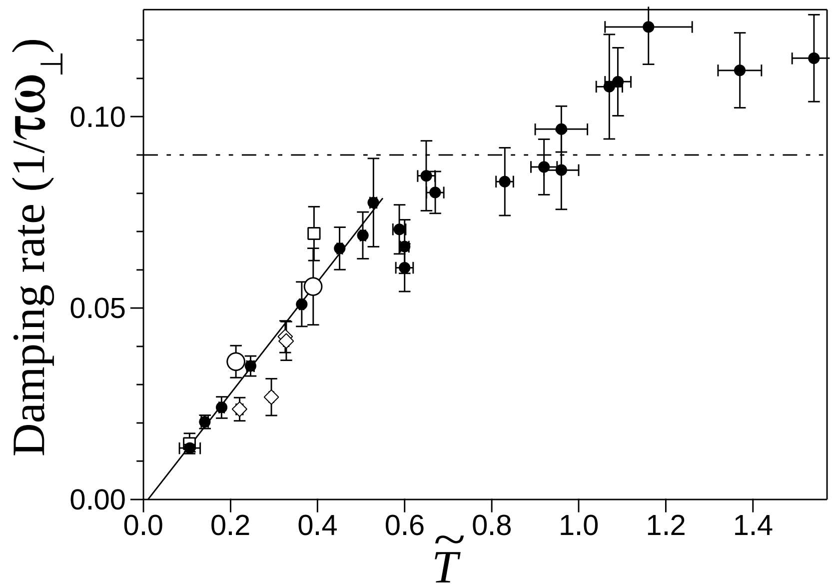
$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\frac{1}{mn}\vec{\nabla}P - \frac{1}{m}\vec{\nabla}V$$

$$\omega = \sqrt{\frac{10}{3}}\omega_\perp$$

Damping of Collective Excitations





$T/T_F = (0.5, 0.33, 0.17)$

$\tau\omega$: decay time $\times$ trap frequency

Kinast et al. (2005)

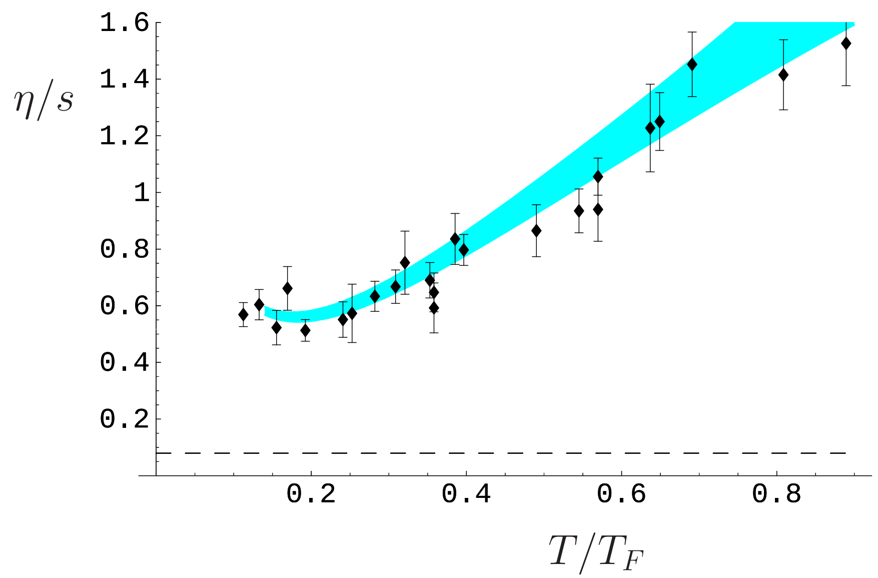
Viscous Hydrodynamics

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{\eta}{2}\int d^3x \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \zeta \int d^3x\, (\partial_i v_i)^2 - \frac{\kappa}{T}\int d^3x\, (\partial_i T)^2$$

Shear viscosity to entropy ratio (assuming $\zeta = \kappa = 0$)

$$\frac{\eta}{s} = \frac{3}{4}\xi^{\frac{1}{2}}(3N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\frac{\bar{\omega}}{\omega_\perp}\frac{N}{S}$$



Schaefer (2007), see also Bruun, Smith, Gelman et al.

Kinetic Theory

Quasi-Particles: Kinetic Theory

$$T_{ij} = \int d^3p \, \frac{p_i p_j}{E_p} f_p,$$

Boltzmann equation

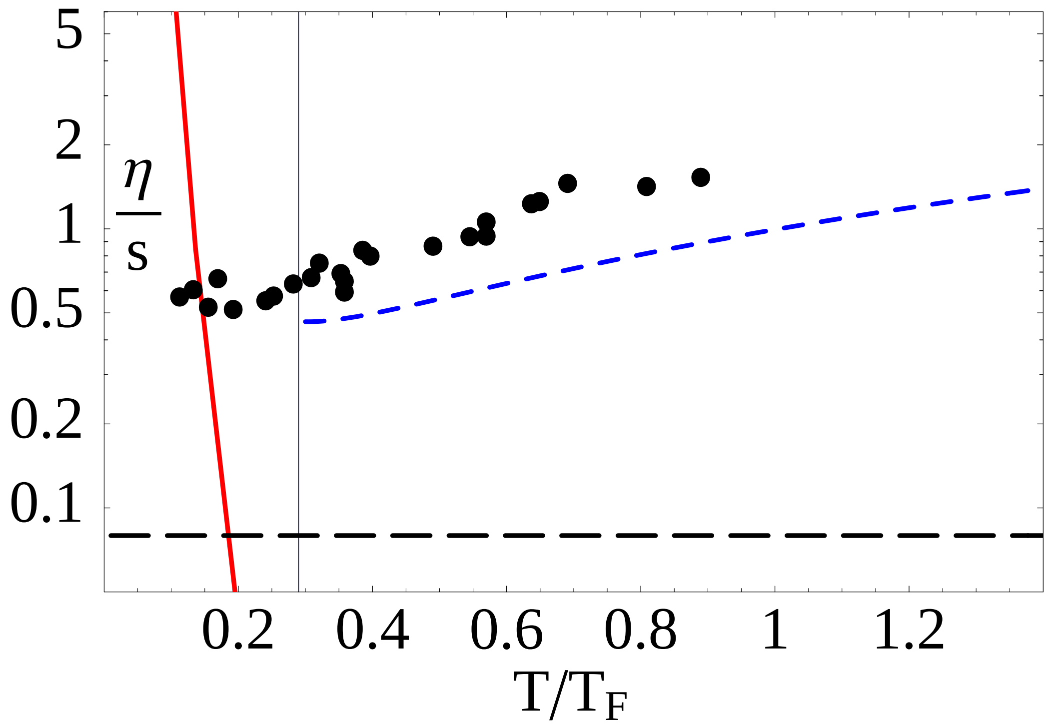
$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

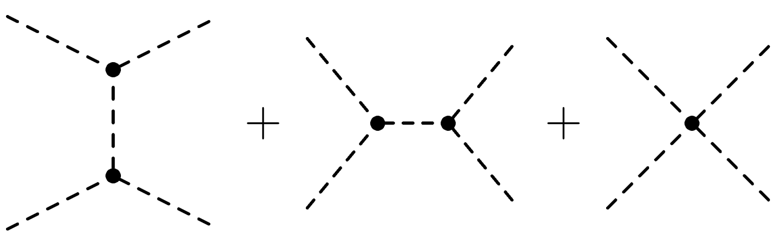
Linearized theory (Chapman-Enskog): $f_p = f_p^0 (1 + \chi_p / T)$

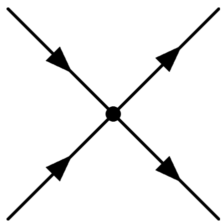
$$\eta \geq \frac{\langle \chi | X \rangle^2}{\langle \chi | C | \chi \rangle} \qquad \langle \chi | X \rangle = \int d^3p \, f_p^0 \, \chi_p \, p_{ij} v_{ij}$$

$$v_{ij} = v^2 \delta_{ij} - 3 v_i v_j$$

Low T: Phonons   High T: Atoms







$$\frac{\eta}{s} \sim \left(\frac{T_F}{T}\right)^8$$

$$\frac{\eta}{s} \sim \left(\frac{T}{T_F}\right)^{3/2} \log\left(\frac{T}{T_F}\right)^{-1}$$

Rupak, Schaefer (2007)

Outlook

Transport near T_c: Relation to Viscosity Bound Conjecture?

Other uses of conformal symmetry? OPE? Braaten, Platter (2008)

AdS/Cold Atom correspondence? Son (2008), Balasubramanian & McGreevy (2008)