Phase transitions, instantons and B mesons

in AdS/CFT with flavor

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work in collaboration with J. Babington, Z. Guralnik, I. Kirsch (HU Berlin),
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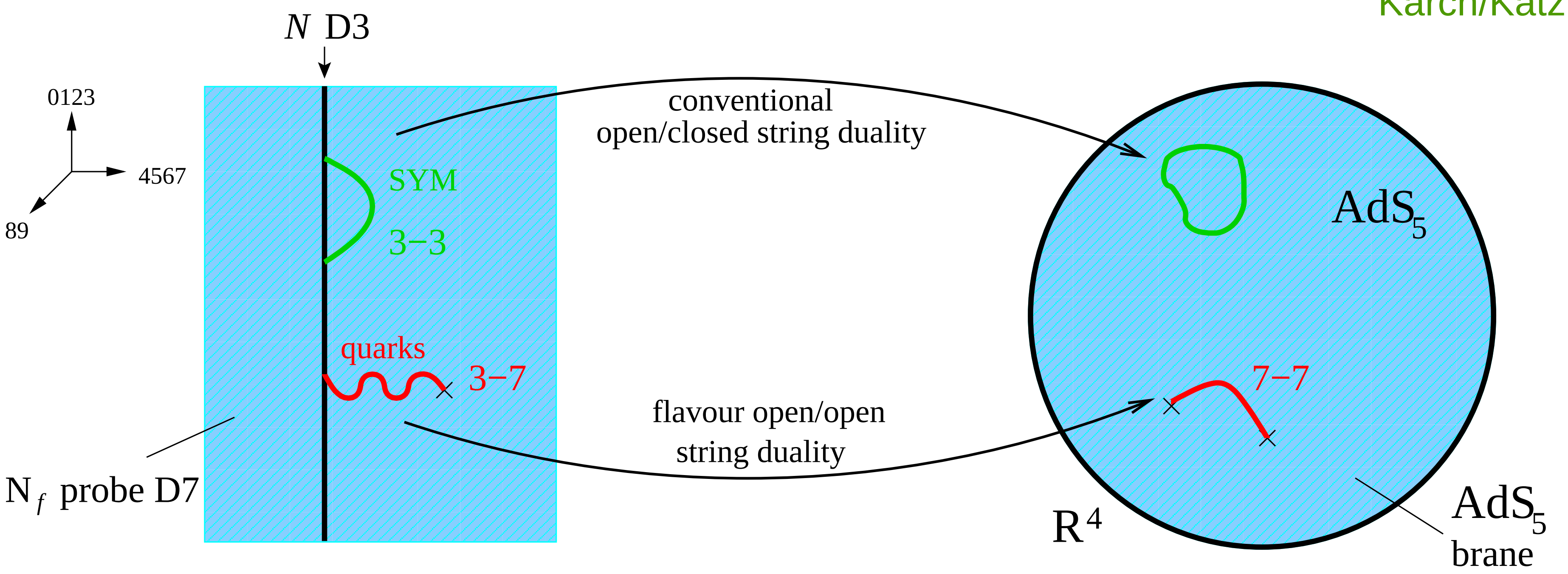
Adding probe D7-branes to AdS/CFT

Karch/Katz, 2002



limit: $N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

Duality acts twice!

4d $\mathcal{N}=4$ SU(N) Super Yang-Mills theory coupled to 4d $\mathcal{N}=2$ hypermultiplet	$\longleftrightarrow$	type IIB SUGRA on $AdS_5 \times S^5$ + Dirac-Born-Infeld theory on $AdS_5 \times S^3$

D7 probe in deformed backgrounds

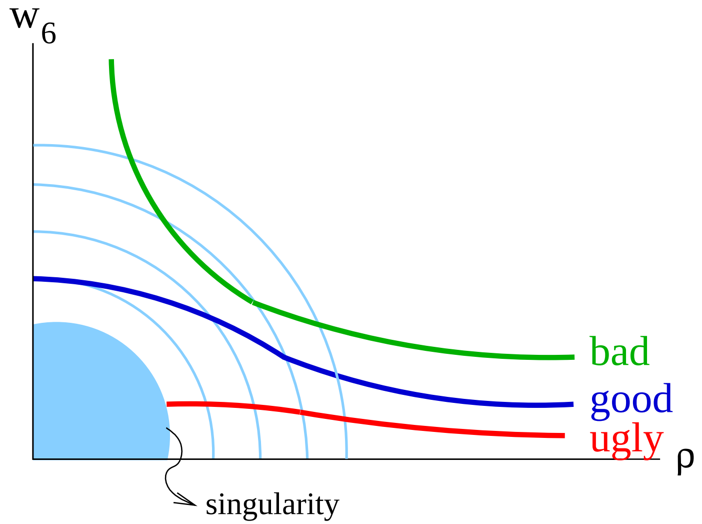
Combine the addition of a D7 brane probe

with the deformation of the $AdS_5 \times S^5$ space
(holographic RG flow to IR theory without supersymmetry)

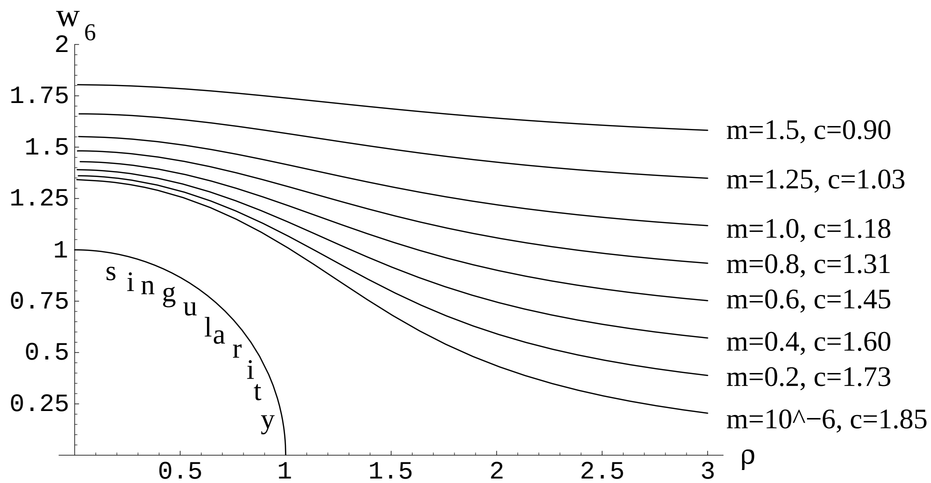
$\Rightarrow$ gravity dual description of spontaneous chiral symmetry breaking by quark condensate and Goldstone bosons

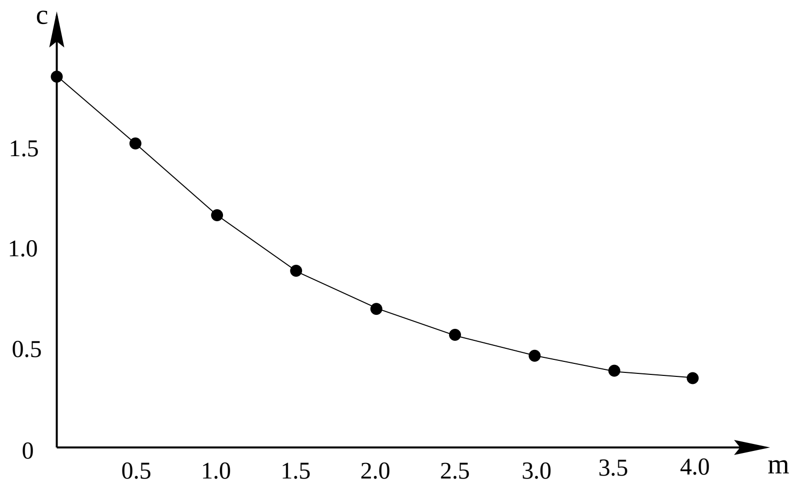
hep-th/0306018, Phys. Rev. D **69** (2004) 066007
with J. Babington, N. Evans, Z. Guralnik, I. Kirsch

Different possibilities for solutions of the D7-brane equations of motion:



Regular solutions in the Constable-Myers (CM) background (b=1):





Numerical results:

screening effect: regular solutions terminate before reaching the singularity!

spontaneous breaking of $U(1)_A$ symmetry: in the limit $m \to 0$ we have $c \neq 0$

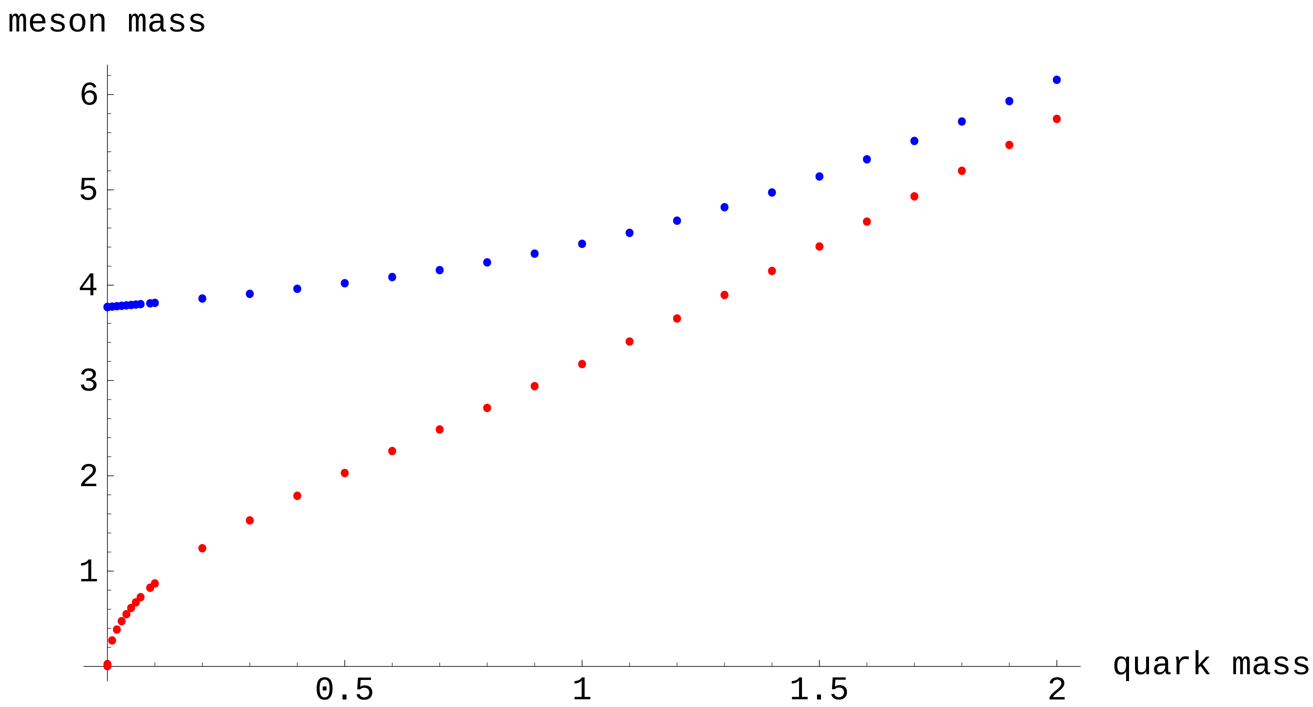
Meson spectrum and large N Goldstone boson (η')

Spontaneous chiral symmetry breaking (via a condensate in the $m \to 0$ limit)
$\Rightarrow$ expect a Goldstone boson in the meson spectrum

Consider fluctuations

$$\delta w_5 = f(\rho)\sin(k \cdot x)\,, \qquad \delta w_6 = h(\rho)\sin(k \cdot x)$$

around $w_5 = 0$, $w_6 = w_6(\rho)$, Meson mass $M^2 = -k^2$



Flavour in the AdS Black Hole geometry

Consider $\mathcal{N} = 4 \; SU(N)$ SYM at finite temperature (Witten, 1998)

The dual string theory background is the Euclidean AdS-Schwarzschild solution defined by the metric (in an appropriate coordinate system)

$$ds^2 = \left(w^2 + \frac{b^4}{4w^2} \right) d\mathbf{x}^2 + \frac{(4w^4 - b^4)^2}{4w^2(4w^4 + b^4)} d\tau^2 + \frac{1}{w^2} \sum_{i=1}^{6} dw_i^2$$

with radial coordinate $w^2 = \rho^2 + w_5^2 + w_6^2$

b a deformation parameter, τ periodic (period $\pi b = T^{-1}$)
horizon: S^1 collapses at $w = \frac{1}{2} b$

AdS Black Hole: Phase transition

Babington, J.E., Evans, Guralnik, Kirsch 2003

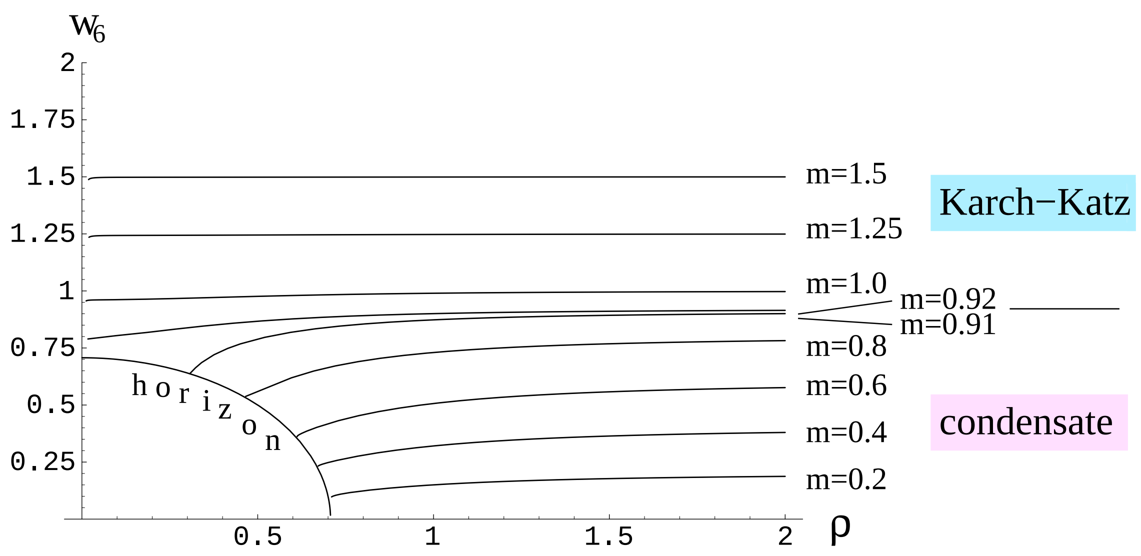
For the regular solutions the D7-brane either ends at the horizon,

$$w^2 = w_6^2 + \rho^2 = \frac{1}{2}b^2 \qquad (S^1\text{collapses})$$

or ends at a point outside the horizon,

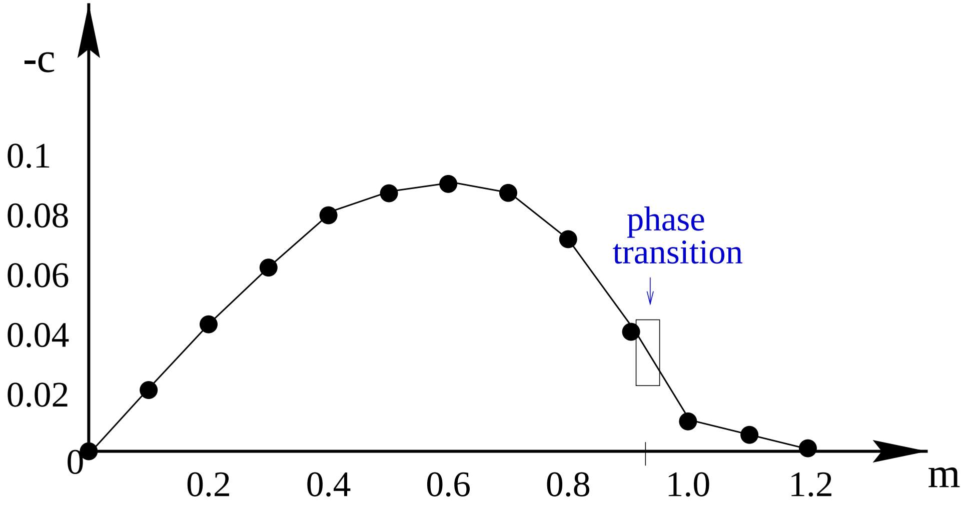
$$\rho = 0, w_6^2 \geq \frac{1}{2}b^2 \qquad (S^3\text{collapses})$$

⇒Two classes of regular solutions in the AdS black hole background:



Condensate at finite temperature

Condensate c versus quark mass m plot, $m \sim \frac{1}{T}$:



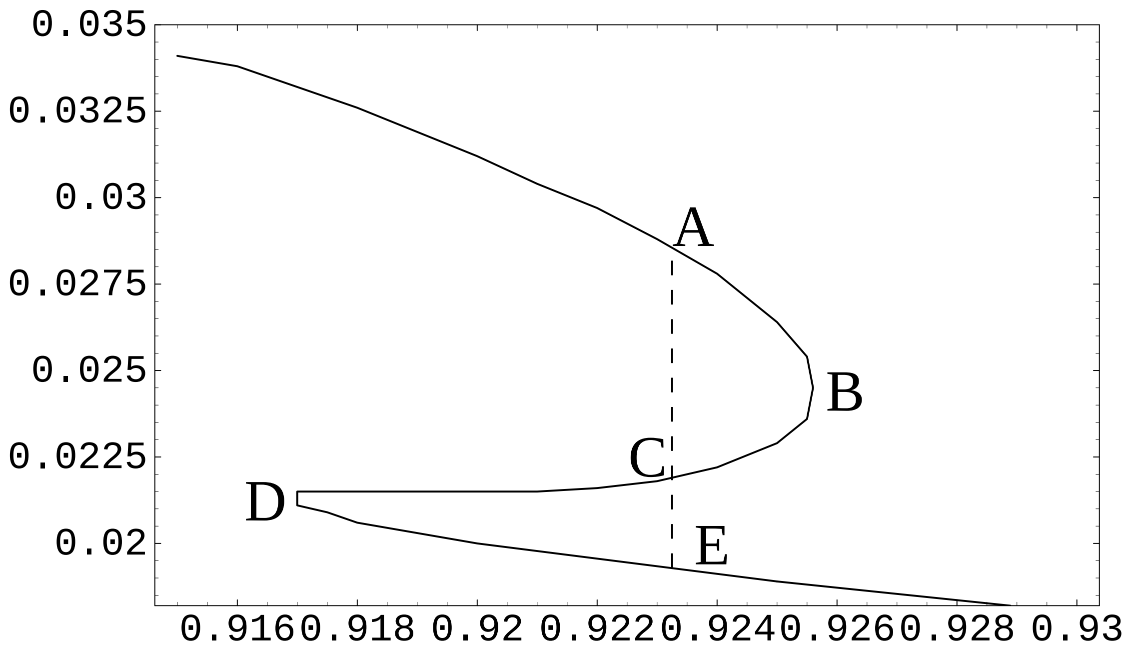
phase transition expected at $m_c \approx 0.92$

no condensate for $m = 0$

(no spontaneous chiral symmetry breaking)

Less symmetric phase at $T_c \leq T \leq \infty$

Phase transition



First order phase transition in type II B AdS black hole background

Ingo Kirsch, PhD thesis 2004

$N_f = 2$ in $\mathcal{N} = 2$ SUSY theory

J.E., Große, Guralnik 0502224

Duality:

Field theory: Higgs branch of $\mathcal{N} = 2$ theory with $N_f = 2$ fundamental hypermultiplets

Dual supergravity configuration: $SU(2)$ instanton configuration on two coincident D7 brane probes in $AdS_5 \times S^5$ background

In particular:

Higgs VEV $\Leftrightarrow$ Instanton size

Gravity dual of Higgs branch with flavour

D7 brane action: $S = S_{WZ} + S_{DBI}$

$$S_{WZ} = \frac{T_7(2\pi\alpha')^2}{4} \int_{D7} C|^{(4)}_{PB} \wedge \mathrm{tr}(F \wedge F)$$

At leading order in α', S_{WZ} and S_{DBI} combine to give

$$S = -T_7(4\pi\alpha')^2 \int d^4x\, d^4y\, \frac{r^4}{4g_s R^4} F_-^2$$

$F^-_{mn} = \frac{1}{2}(F_{mn} - \frac{1}{2}\epsilon_{mnrs}F_{rs})$ $\qquad r^2 = y^2 + m^2$

x, μ: 0123 directions (D3/D7), $\quad y, m$: 4567 directions (D7 only), $\quad m$: position of D7 in 8 direction

Field strengths which are self-dual with respect to the flat metric $dy^m dy^m$ manifestly solve the equations of motion $(F^-_{mn} = 0)$

Fluctuation spectrum for vector mesons

Fluctuation spectrum for vector mesons:

Simplest non-Abelian ansatz for fluctuations about the instanton background:

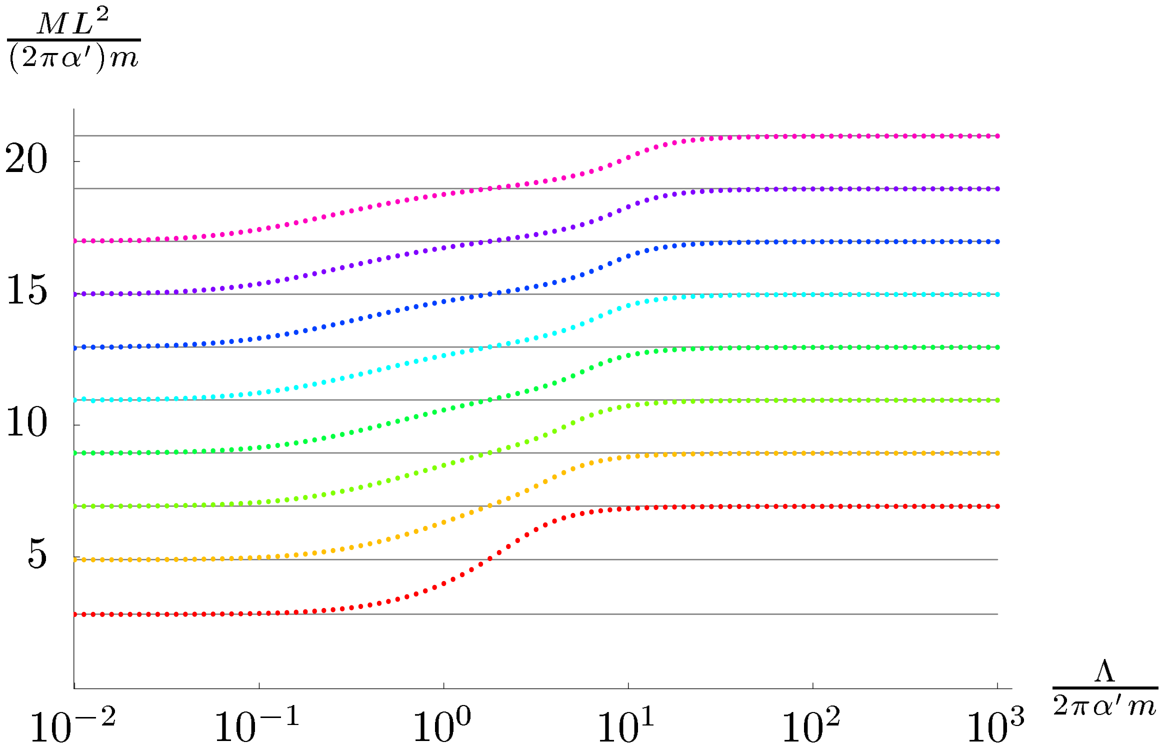
$$A_\mu{}^{(a)} = \xi_\mu(k) f(y) e^{ik^\mu x_\mu} \tau^a, \qquad M^2 = -k^\mu k_\mu$$

τ^a: Pauli matrices

Asymptotically, in dual gauge theory this corresponds to
. matrix elements of the $SU(2)$ flavour current

. x, μ: 0123 directions (D3/D7), y, m: 4567 directions (D7 only)

Fluctuation spectrum for vector mesons - Spectral flow



The spectrum is shifted by two levels between zero and infinite instanton size!

$N_f = 2$ in black hole background

Apreda, J.E., Evans, Guralnik 0504151

Deformed backgrounds:

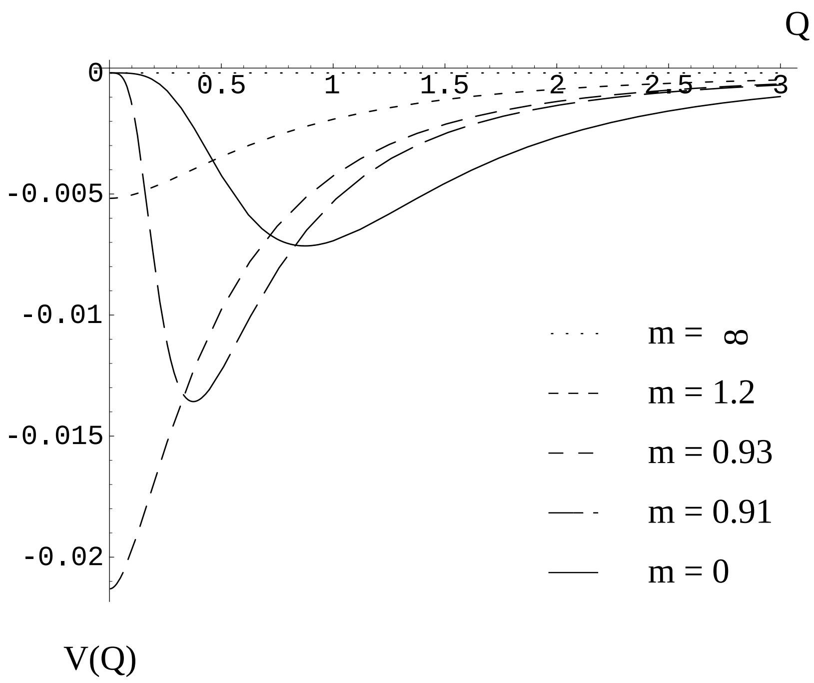
$S_{WZ} + S_{DBI}$ for the D7 probes evaluated on a static instanton configuration

$\longrightarrow$ potential for the squark bilinear (Higgs VEV).

$$S_{D7} = -\int d^4x V(Q)$$

$$V(Q) = T_7 \frac{(2\pi\alpha')^2}{2} \left(\frac{1}{g_s} \int d^4y \, C^{(4)}_{0123} \, \epsilon_{mnrs} \, \mathrm{Tr} \, F_{mn} F_{rs} \right.$$
$$\left. - \frac{1}{2} \int d^4y \sqrt{-\mathrm{det} G} \, \mathrm{Tr} F^{mn} F_{mn} \right)$$

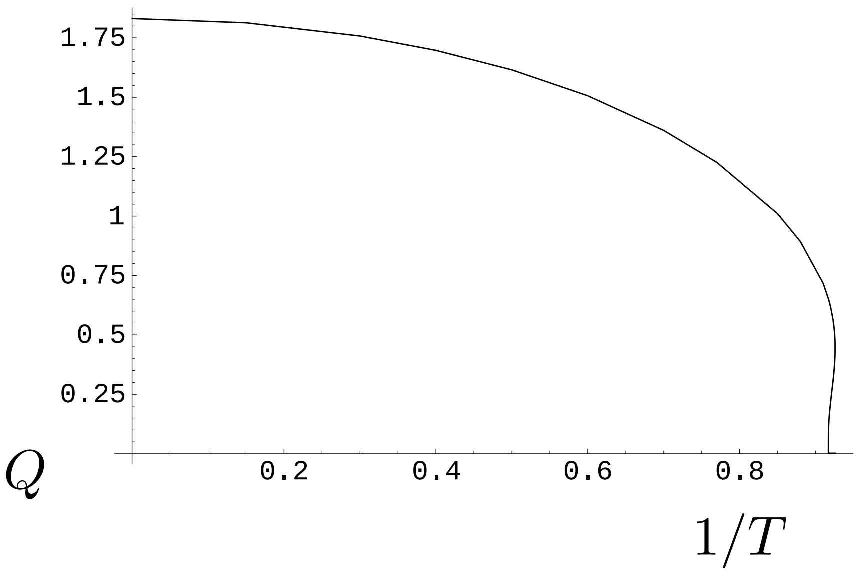
$N_f = 2$ in black hole background

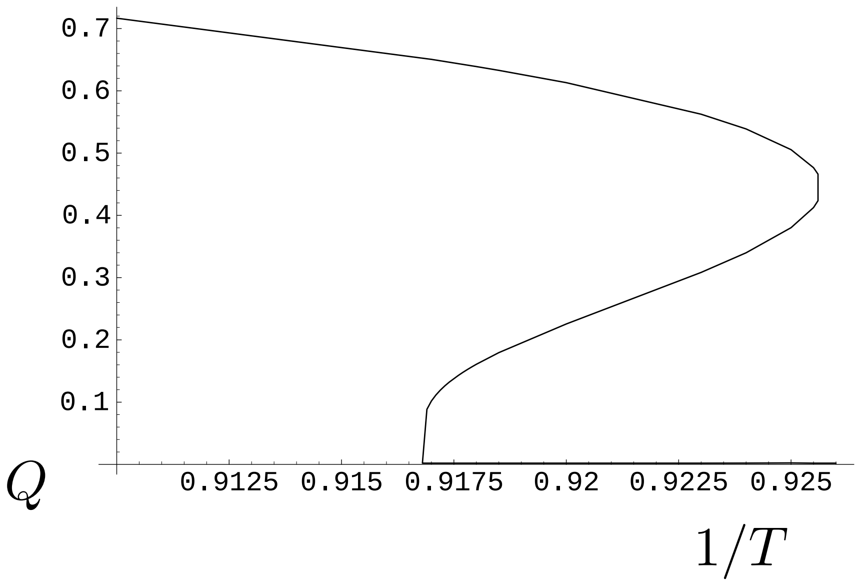


Higgs Potential for different values of $m_{quark} \propto 1/T$

Squark VEV Q which minimizes Higgs potential

$\longrightarrow$ order parameter for the 1st order phase transition





The potential for the squark VEV is given within the dual gravity theory by the action evaluated on the $SU(2)$ instanton configuration.

Further uses of Higgs potential

- Chemical potential

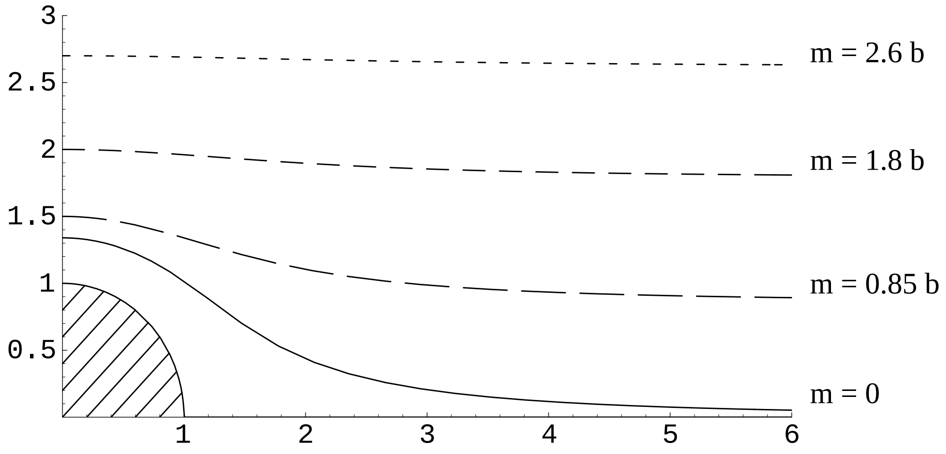
  A^0 component of spurious gauge field associated with $SU(2)$ symmetry acquires a VEV
  $\Rightarrow$ instability on both field theory and gravity side (potential negative)

- Stability of brane embeddings

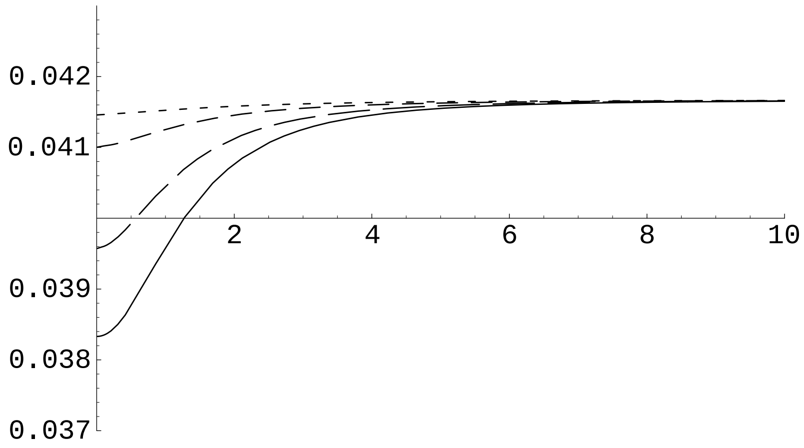
  runaway behaviour of Higgs VEV for a given supergravity background would signal instability

Higgs potential for Constable-Myers deformation

w_6 vs. ρ



$V(Q)$ vs. Q



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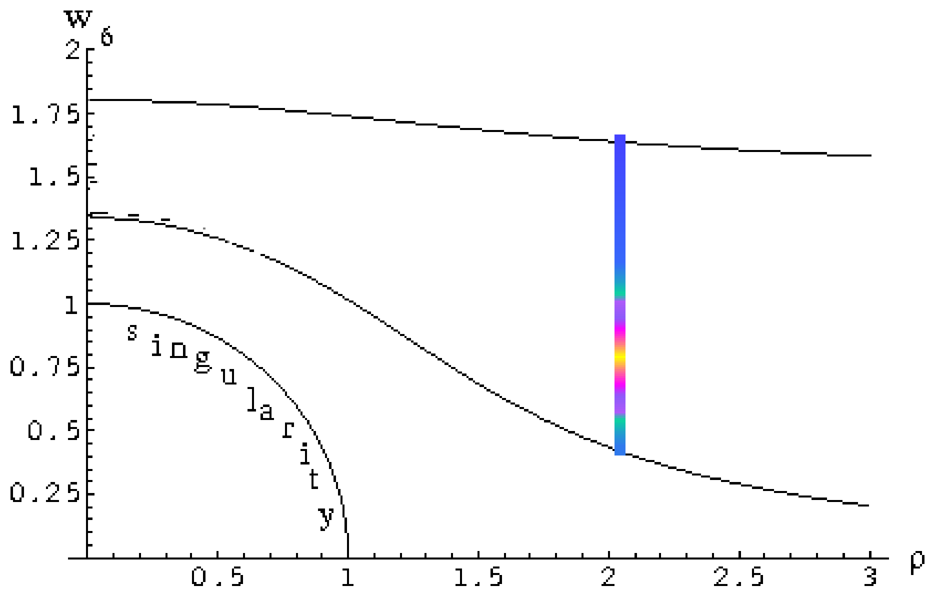
Apreda, J.E., Evans 0509219

Heavy-light mesons J.E., Evans, Große 2006

Consider string stretched between two D7 branes corresponding to different quark masses



B mesons

Polyakov string action

$$S = -\frac{T}{2}\int d\sigma d\tau G_{\mu\nu}(-\dot{X}^{\mu}\dot{X}^{\nu} + X'^{\mu}X'^{\nu})$$

Constraints:

$$G_{\mu\nu}\dot{X}^{\mu}X'^{\nu} = 0, \qquad G_{\mu\nu}(\dot{X}^{\mu}\dot{X}^{\nu} + X'^{\mu}X'^{\nu}) = 0$$

Identify $\sigma = w_6$, integrate over σ

Action:

$$S = -\frac{T}{2}\int d\tau \left[f(\rho)\dot{x}^2 - g(\rho)\dot{w}_i^2 + g(\rho)\right]$$

$$f(\rho) = \rho^2 L + \frac{1}{3}L^3, \qquad g(\rho) = \frac{1}{\rho}\mathrm{ArcTan}(L/\rho)\,.$$

Legendre transform:

$$p_\alpha = \frac{\delta\mathcal{L}}{\delta\dot{x}^\alpha}$$

B mesons

Constraint:

$$\frac{1}{T^2 f(\rho)} p_x^2 + \frac{1}{T^2 g(\rho)} p_w^2 + g(\rho) = 0$$

Simple modification of $E^2 - p^2 = m^2$ with effective mass depending on the ρ position of the string

Quantum mechanical operator substitutions: $p \rightarrow -i\nabla$ acting on scalar field

$$\left[\Box_x^2 - \frac{f(\rho)}{g(\rho)} \nabla_w^2 + T^2 g(\rho) f(\rho) \right] \phi = 0$$

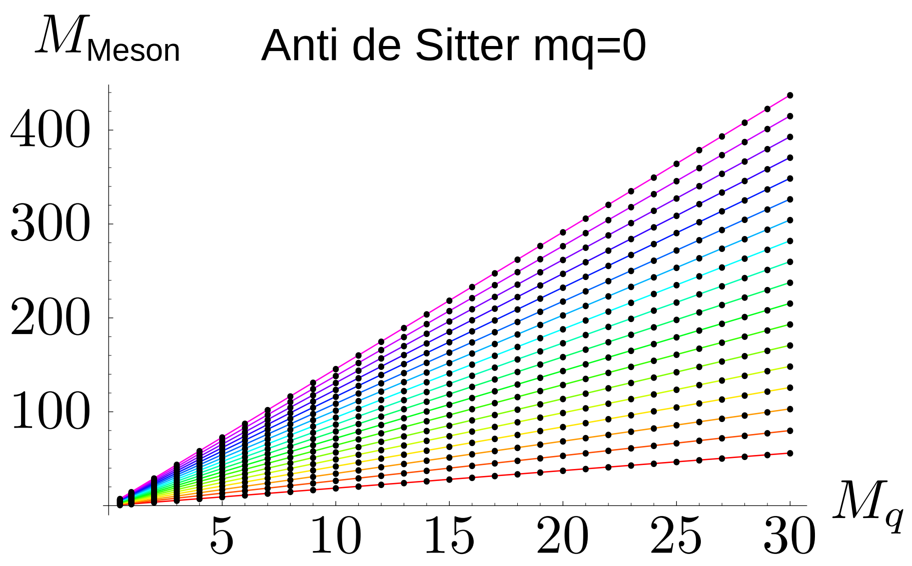
Asymptotic behaviour:

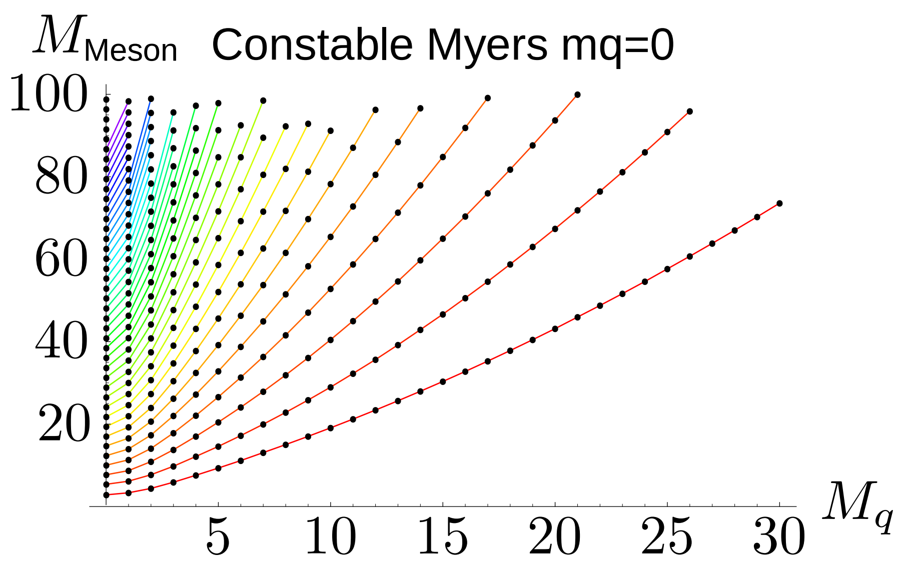
$$\phi = m_{hl} + \frac{c_{hl}}{\rho^2} + \ldots$$

Fluctuations

$$\phi = h(\rho) e^{ik.x}, \qquad k^2 = -M^2$$

Heavy-light meson masses





B mesons

Input: Masses of ρ and Υ mesons

ρ: $(\bar{u}d)$ vector meson, $M_\rho = 770$ MeV

Υ: $(\bar{b}b)$ vector meson, $M_\Upsilon = 9.4$ GeV

Result:

Our model gives $M_B = 6.2$ GeV for B meson

B: $(\bar{b}u)$ pseudoscalar

Experimental result: $M_B = 5.3$ GeV

Conclusion

- We have given a holographic description of chiral symmetry breaking by a quark condensate and of light mesons.

- An instanton on the D7 brane probe leads to a spectral flow of the meson spectrum on the Higgs branch.

- The gauge/gravity dual description predicts interesting new features in finite temperature field theories, such as a first order transition at high temperature

- The calculation of the Higgs potential provides an order parameter for the phase transition, a description of the chemical potential and a tool to clarify stability issues.

- We have constructed a simple model for studying heavy-light mesons.

The Constable-Myers deformation

The Myers-Constable background is given by the metric

$$ds^2 = H^{-1/2}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta/4} dx_4^2 + H^{1/2}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{(2-\delta)/4} \frac{w^4-b^4}{w^4}\sum_{i=1}^{6} dw_i^2,$$

where

$$H = \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta} - 1 \qquad (\Delta^2+\delta^2=10)$$

and the dilaton and four-form

$$e^{2\phi} = e^{2\phi_0}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{\Delta}, \qquad C_{(4)} = -\frac{1}{4}H^{-1} dt\wedge dx\wedge dy\wedge dz$$

This background has a singularity at $w=b$