

Deconfining phase of SU(2) Yang-Mills thermodynamics

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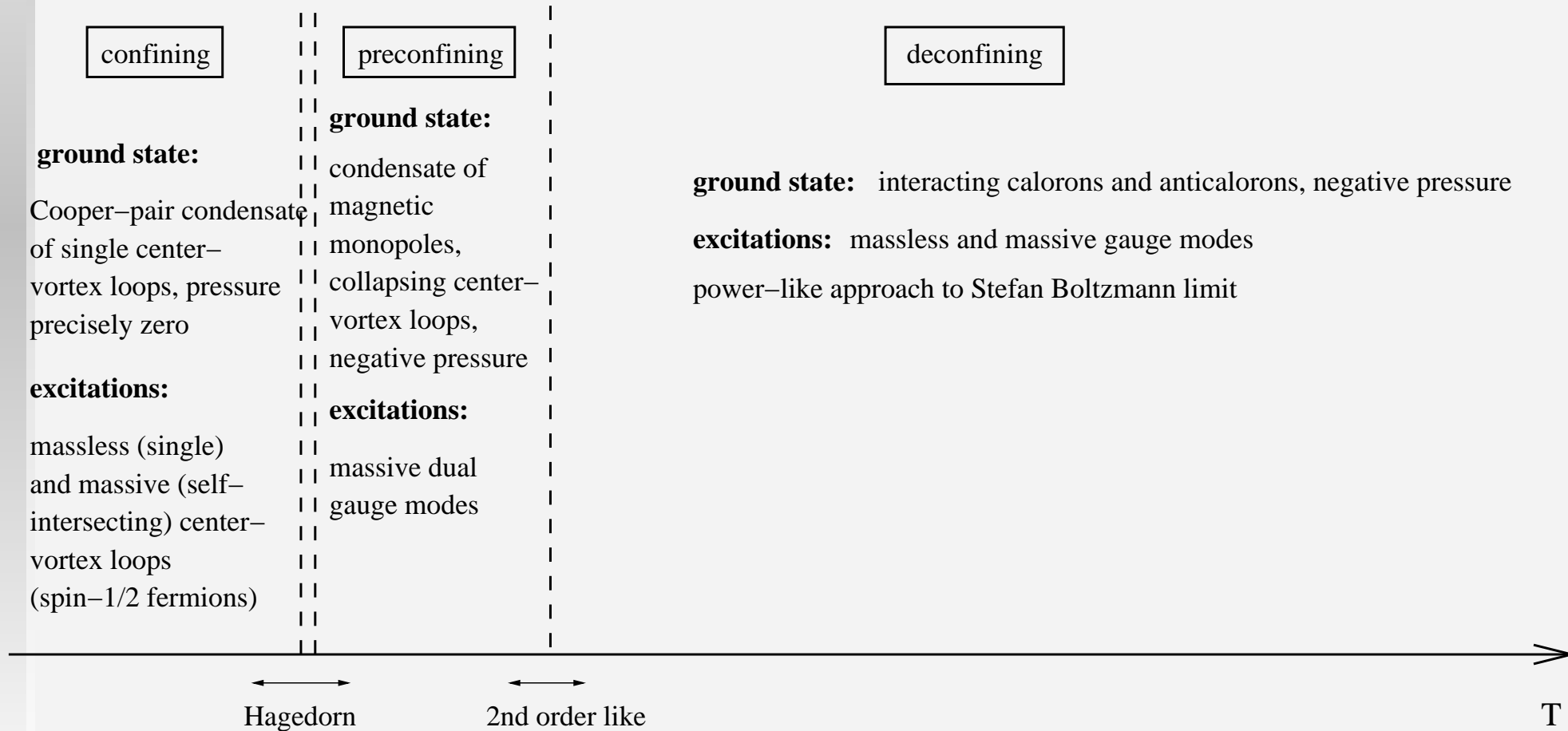


outline

- ▶ a nonperturbative ground state at high temperature: coarse-grained, interacting calorons and anticalorons
- ▶ dynamical gauge-symmetry breaking and evolution of coupling
- ▶ thermodynamical quantities
- ▶ polarization tensor
- ▶ application: $SU(2)_{\text{CMB}}$, fate of the photon, and black-body spectrum
- ▶ summary

phase diagram: SU(2) YMTD

[RH 2005]



nonperturbative ground state

basic idea:

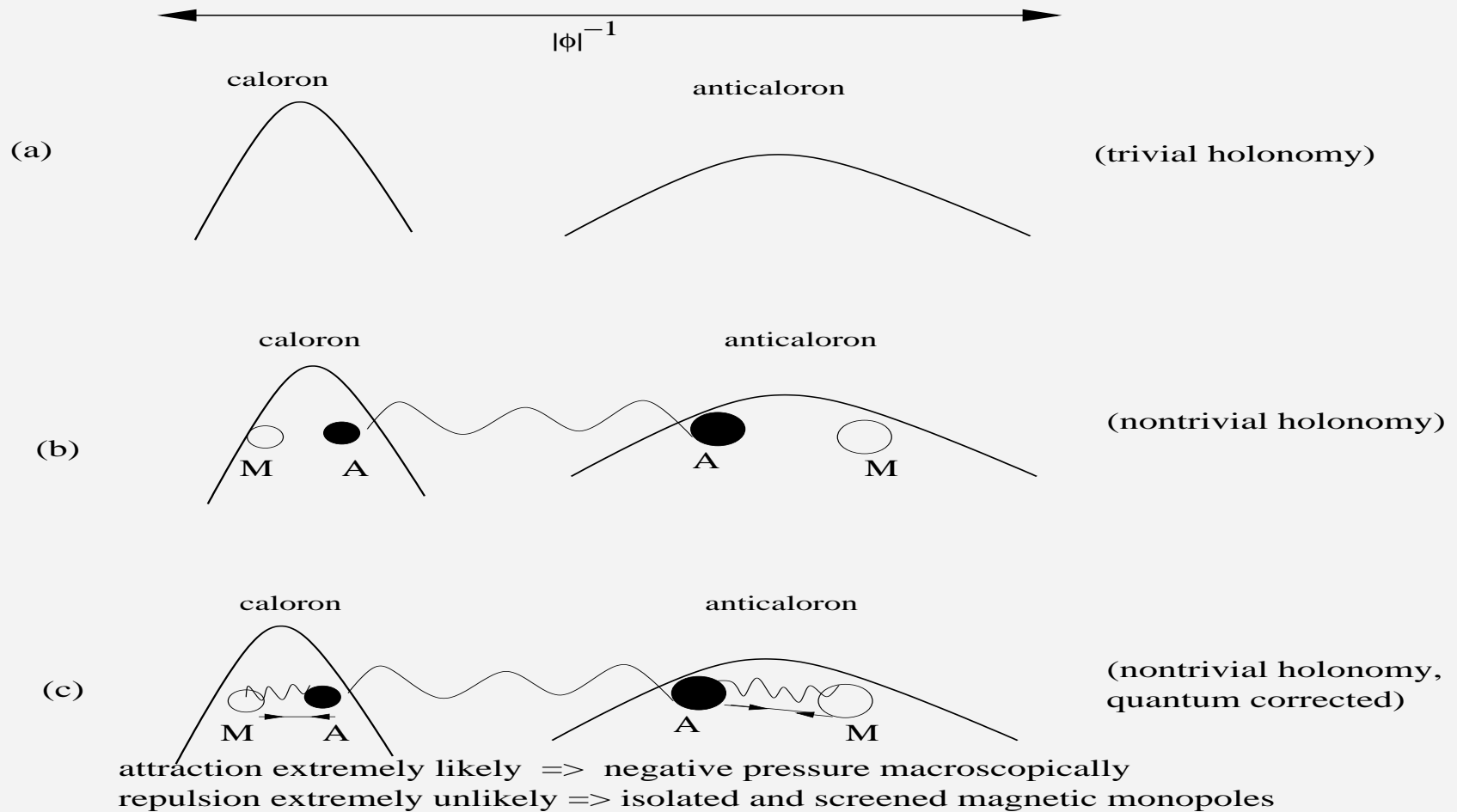
(i) coarse-grain free topological defects ((anti)calorons)

⇒ inert adjoint scalar field $\phi^a(\tau, T)$

(ii) introduce interactions (trivial-topology fluctuations)

in terms of *pure-gauge* solution to
coarse-grained equation of motion

ground-state physics (picture):



ground-state physics (sketch of technique):

ϕ^a emerges upon spatial coarse-graining

\Rightarrow in given gauge τ -dependence (winding) of phase $\hat{\phi}^a$
classically determined

$\Rightarrow \hat{\phi}^a$ lies in the kernel of diff. operator \mathcal{D} :

$$\hat{\phi}^a(\tau) \in \sum_{\text{HS (anti)caloron}, Q=\pm 1} \text{tr} \int d^3x \int d\rho \frac{\lambda^a}{2} \times$$

$$F_{\mu\nu}((\tau, 0)) \{(\tau, 0), (\tau, \vec{x})\} \times$$

$$F_{\mu\nu}((\tau, \vec{x})) \{(\tau, \vec{x}), (\tau, 0)\} .$$

observation:

RHS is *annihilated* by

$$\mathcal{D} = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$$

and (modulo global g. t.) ambiguities in evaluating RHS match parametrization of \mathcal{D} 's kernel

$\Rightarrow \mathcal{D}$ uniquely determined

moreover:

(anti)caloron BPS

$\Rightarrow \phi^a(\tau)$ BPS

\Rightarrow e.o.m is: $\partial_\tau \hat{\phi} = \pm \frac{2\pi i}{\beta} \lambda_3 \hat{\phi}$

$\Rightarrow \hat{\phi} = C \lambda_1 \exp\left(\pm \frac{2\pi i}{\beta} \lambda_3 (\tau - \tau_0)\right).$

ϕ 's modulus:

coarse-graining over infinite space saturated at length $|\phi|^{-1}$ if existence of Yang-Mills scale Λ and analyticity of RHS BPS equ. assumed

$$\Rightarrow \text{BPS equation } \partial_\tau \phi = \pm i \Lambda^3 \lambda_3 \phi^{-1}$$

$$\Rightarrow |\phi| = \sqrt{\frac{\Lambda^3}{2\pi T}}$$

effect. act. for free, coarse-grained (anti)calorons:

$$S_\phi = \int_0^\beta d\tau \int d^3x \text{tr} \left(\partial_\tau \phi \partial_\tau \phi + \frac{\Lambda^6}{\phi^2} \right) .$$

(in singular gauge, microscopically)

$\Rightarrow \phi$ does not fluctuate statist. nor quantum mech.

minimal coupling of coarse-grained, trivial fluctuations:

$$S_{\text{YM}} = \int_0^\beta d\tau \int d^3x \text{tr} \left(\frac{1}{2} G_{\mu\nu} G_{\mu\nu} + D_\mu \phi D_\mu \phi + \frac{\Lambda^6}{\phi^2} \right) .$$

(perturbative renormalizability! no additive constant in potential!)

$$\Rightarrow D_\mu G_{\mu\nu} = 2ie[\phi, D_\nu \phi]$$

$$\Rightarrow \text{solution } a_\mu^{bg} = \frac{\pi}{e} T \delta_{\mu 4} \lambda_3$$

$$\Rightarrow \rho^{g.s} = 4\pi \Lambda^3 T = -P^{g.s} \neq 0$$

(gluon exchanges induce nontrivial holonomy, creation of monopoles (M) and antimonopoles (A), attractive potential between M and A [[Diakonov et al. 2004](#)])

coarse-grained fluctuations in real time

adjoint Higgs mechanism:

rotation to unitary gauge $\phi = \lambda_3 |\phi|$, $a_\mu^{bg} = 0$

\Rightarrow admissible periodic but singular gauge transformation

\Rightarrow *but* Polyakov loop: electric Z_2 degeneracy

\Rightarrow discussed phase indeed *deconfining*

($SU(2) \rightarrow U(1)$): existence of massless direction!)

mass of off-diagonal fluctuations V_μ^\pm :

$$m_{V^\pm} = 2e|\phi|$$

constraints on quantum fluctuations in loop diagrams:

in physical gauge (unitary-Coulomb) and lowest order:

(i) $|p^2 - m_{V\pm}^2| \leq |\phi|^2$ or $|p^2| \leq |\phi|^2$ (3-vertex, propagation)

(ii) $|(p + q)^2| \leq |\phi|^2$ (4-vertex)

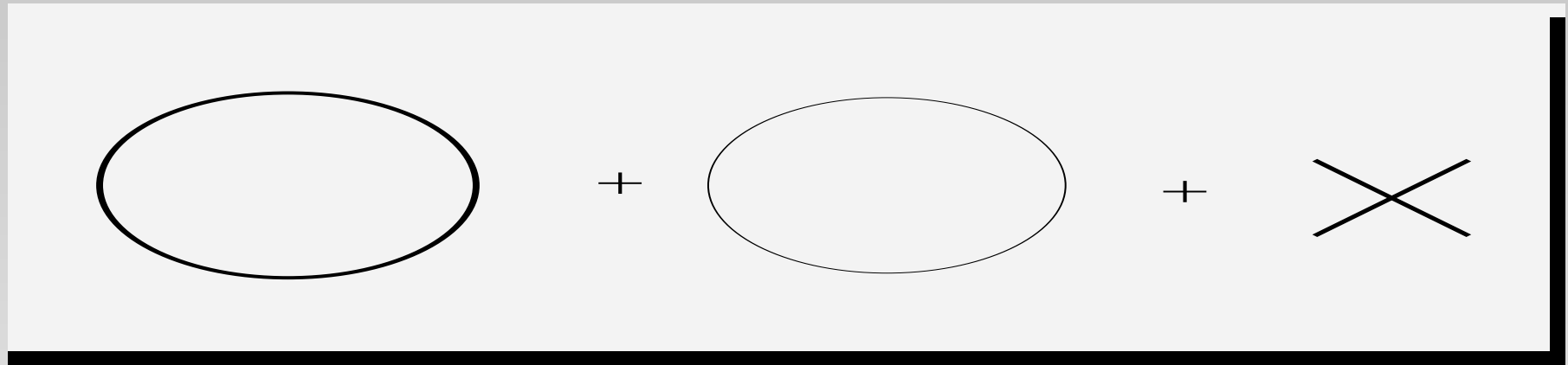
\Rightarrow expansion *terminates* at two-loop level

thermodynamical quantities:

one loop dominates over two loops by a factor $\sim 10^3$

[Herbst, Hofmann, Rohrer 2004; Schwarz, Hofmann, Giacosa 2006]

one-loop pressure P :



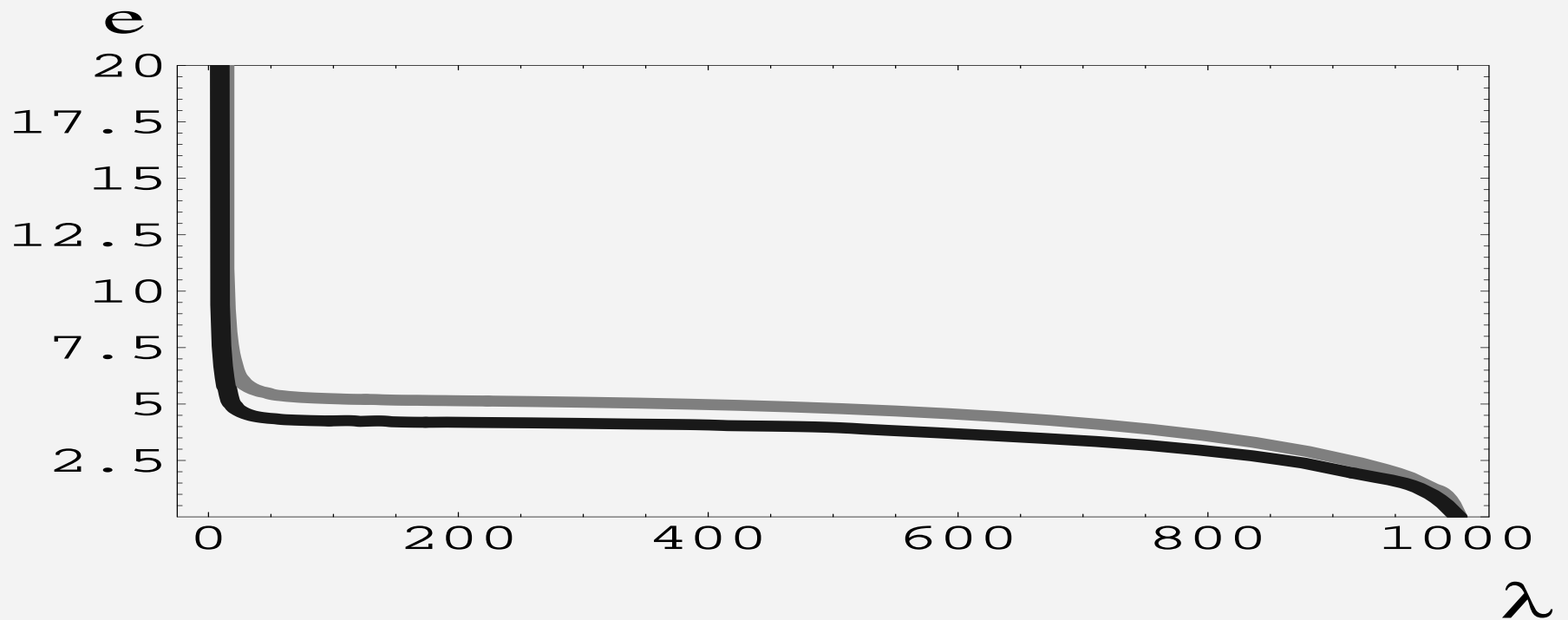
evolution equation for e :

invariance of Legendre transformations

$$\Rightarrow \partial_a \lambda = -\frac{24 \lambda^4 a}{(2\pi)^6} D(2a) \quad \text{where}$$

$$D(a) \equiv \int_0^\infty dx \frac{x^2}{\sqrt{x^2+a^2}} \frac{1}{\exp(\sqrt{x^2+a^2})-1}, \quad a \equiv \frac{m_{V^\pm}}{T}, \quad \lambda \equiv \frac{2\pi T}{\Lambda}.$$

after inversion: $\lambda(a) \rightarrow a(\lambda) \Rightarrow$



► IR-UV decoupling

► logarithmic pole: $e \sim -\log(\lambda - \lambda_c)$

⇒ total screening of monopoles: $m_{\text{mon}} \propto \frac{T}{e}$

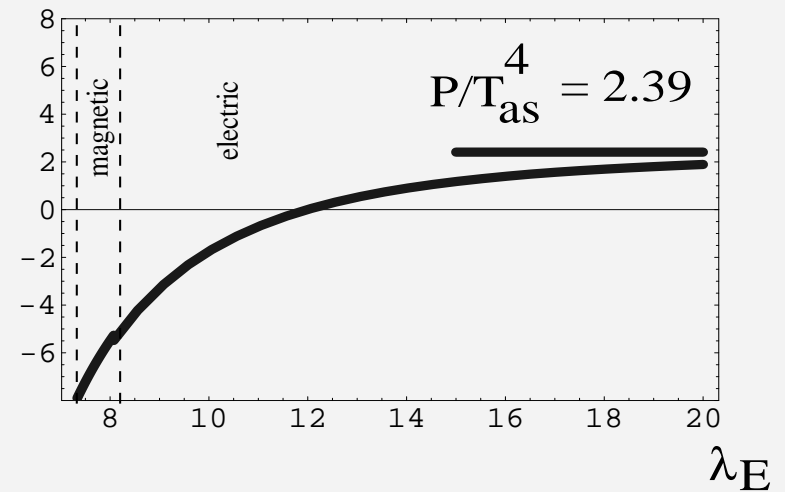
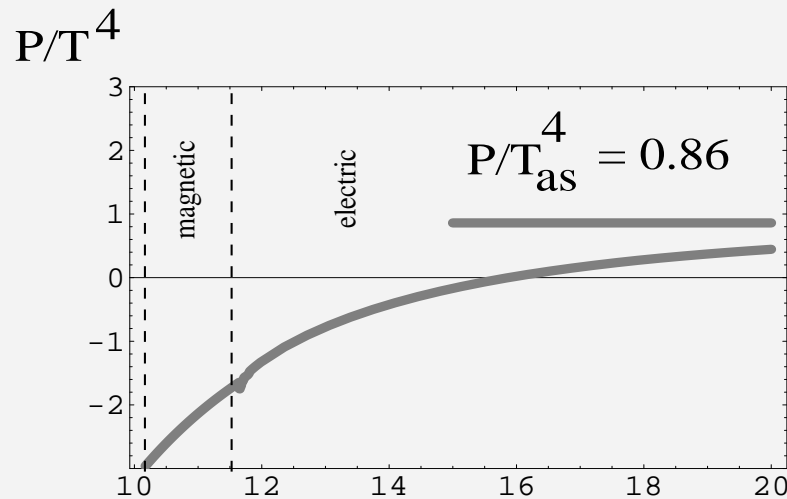
⇒ instability towards large (anti)caloron holonomy,
condensation of monopoles

thermodynamical quantities

pressure (infrared sensitive):

SU(2)

SU(3)

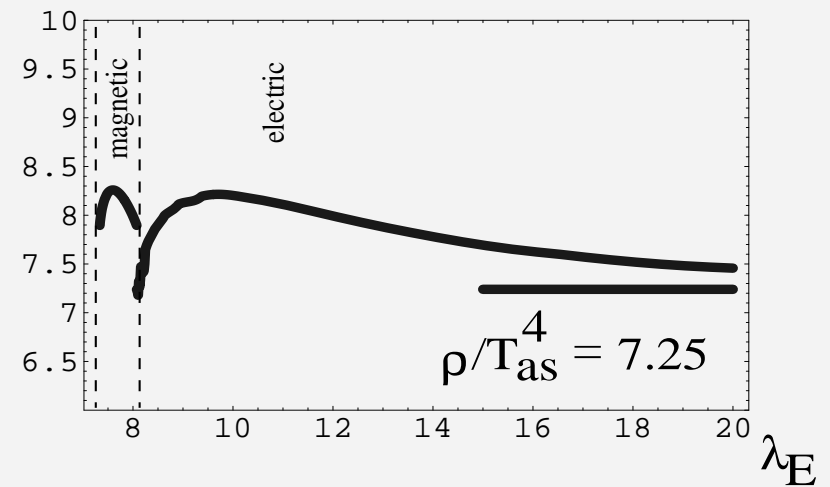
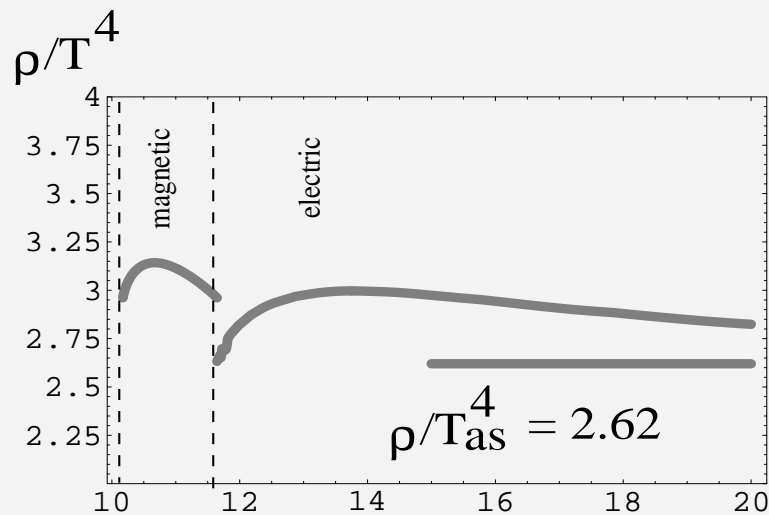


thermodynamical quantities

energy density (infrared sensitive):

SU(2)

SU(3)

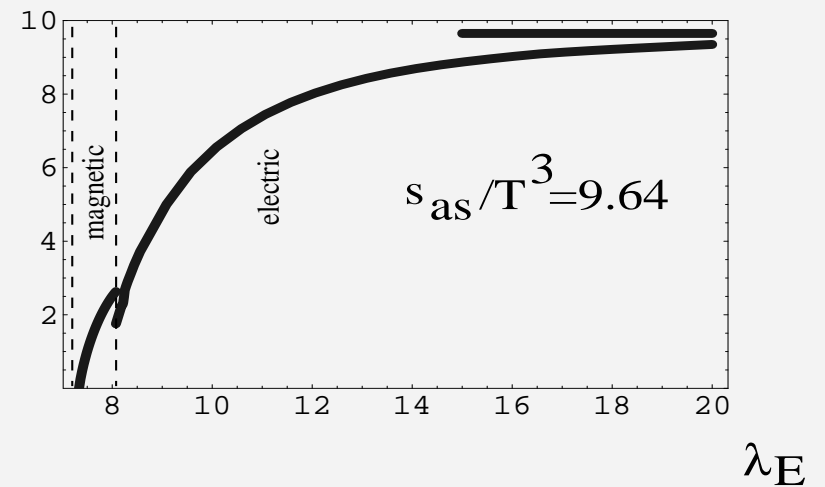
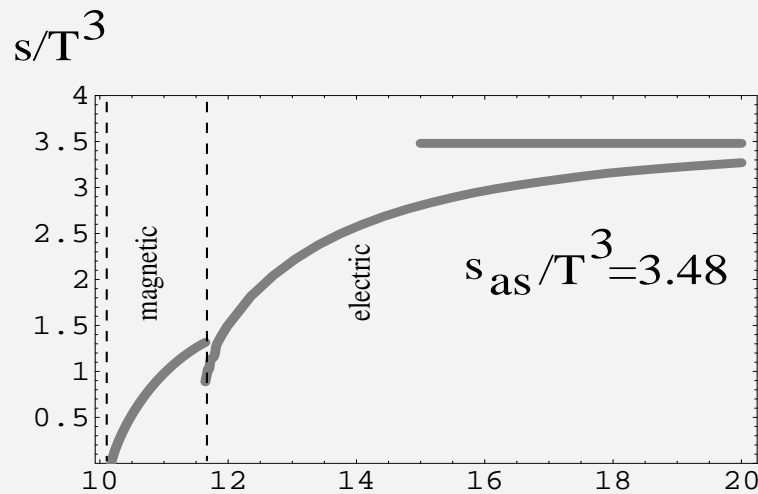


thermodynamical quantities

entropy density (infrared safe): $sT = P + \rho$

SU(2)

SU(3)



application: $SU(2)_{\text{CMB}}$

hypothesis:

$U(1)_Y$ today coincides with $SU(2)_{\text{CMB}}$

$\Rightarrow SU(2)_{\text{CMB}}$ at boundary deconf.-preconf. phase

$\Rightarrow \Lambda_{\text{CMB}} \sim 10^{-4} \text{ eV}$

$\Rightarrow \rho_{\text{CMB}}^{g.s.} \sim 0.05\% \Lambda_{\text{cosmo}}$

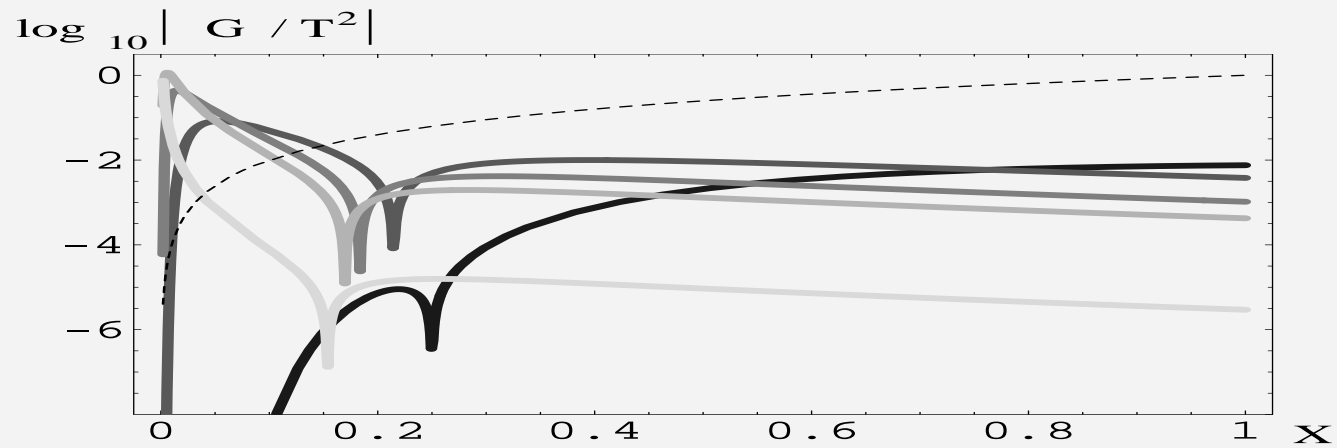
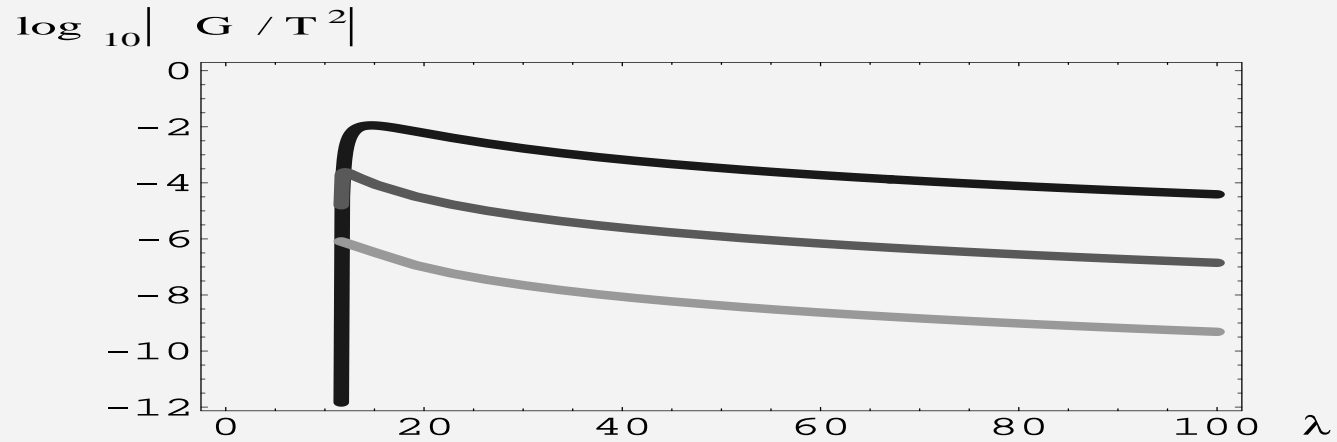
\Rightarrow with slow-roll axion field a , $m_a \sim 10^{-36} \text{ eV}$,

\Rightarrow explanation for Universe's dark-energy content

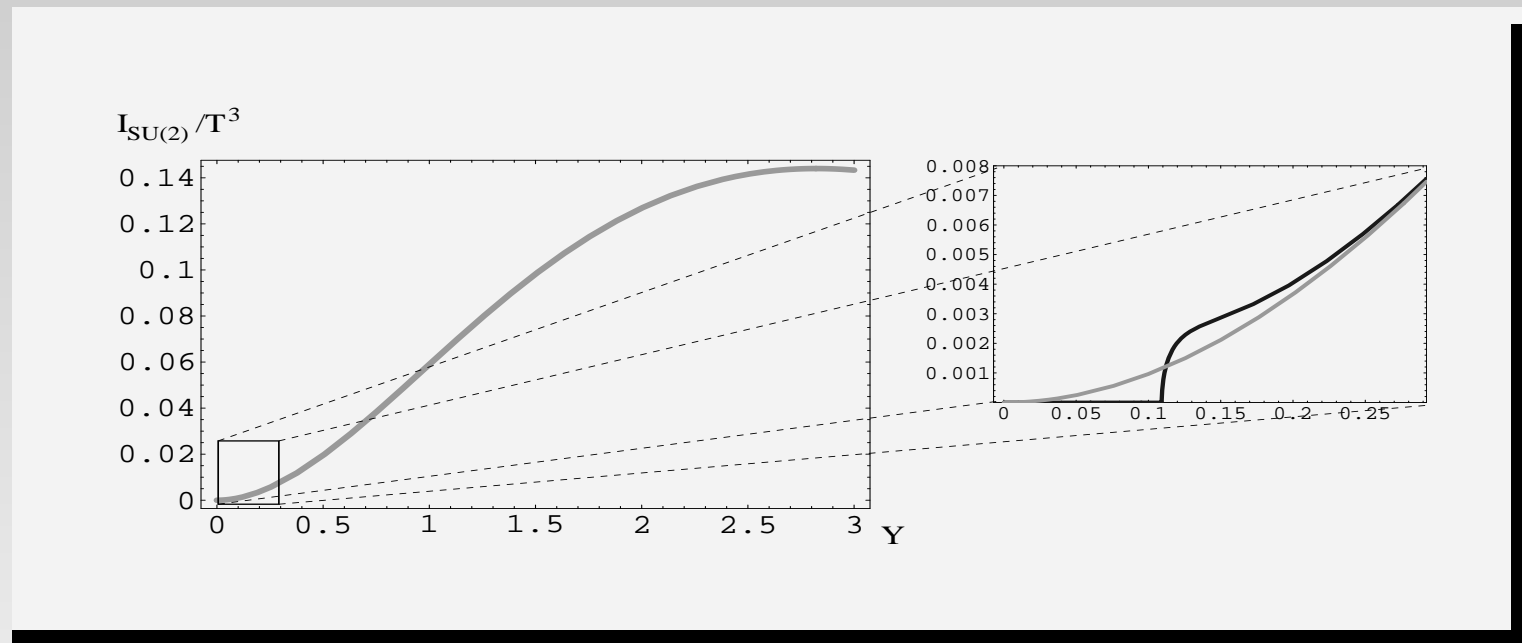
[Frieman et al. 1995; Wetterich 2001; Giacosa, Hofmann 2005]

\Rightarrow photon Meissner massive < 2 billion years

mod. photon propag.: photon's polarization tensor



mod. Planck spectrum at $T = 10$ K.:



\Rightarrow translates into $\frac{\delta T}{T} \sim 10^{-3}$

\Rightarrow matches strength of CMB dipole!

also:

(i) nonexistence of propagating photons for

$$p < 0.1 T \sim 1 \text{ K}$$

possible explanation for old, cold, and dilute H1 clouds

($\rho \sim 1 \text{ cm}^{-3}$, age 50 million years, $T \sim 10 \text{ K}$)

[Brunt,Knee 2001]

(ii) PVLAS experiment: 'vacuum' birefringence induced
by intermediate V^\pm ?

(fluid ^4He boiling point: $T = 4.2 \text{ K} (> 2.7 \text{ K})$)

summary

- ▶ even at high temperature:
nonperturbative ground state composed of
coarse-grained, interacting (anti)calorons ($Q = \pm 1$)
- ▶ physics of coarse-grained excitations
- ▶ thermodynamical quantities
- ▶ $SU(2)_{\text{CMB}}$: photon's fate and polarization tensor
- ▶ implications for the real world