Light-Cone Sum Rules with B-Meson Distribution Amplitudes

Alexander Khodjamirian (University of Siegen)

(with Thomas Mannel and Niels Offen)

Continuous Advances in QCD, FTPI, Minneapolis, May 11-14, 2006

QCD light-cone sum rules (LCSR)

Balitsky, Braun, Kolesnichenko; Braun, Filyanov (1989);
Chernyak, Zhitnisky(1990)

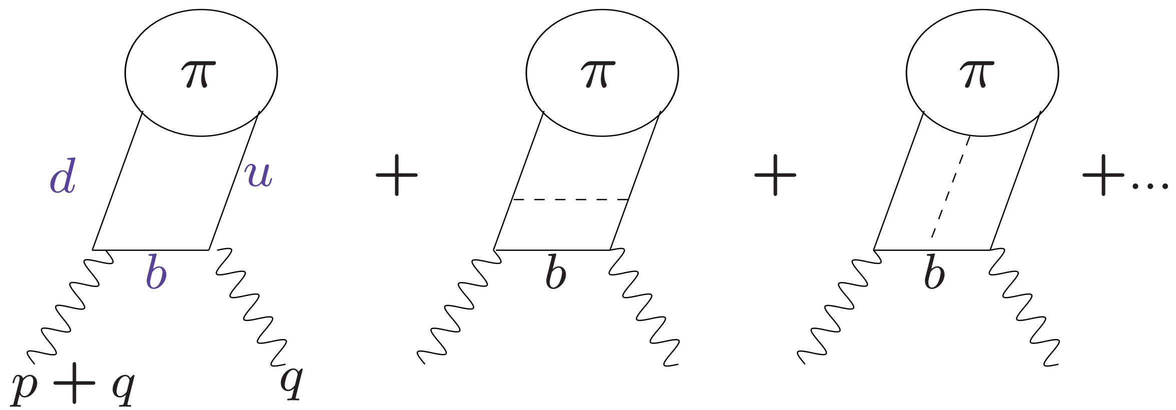
allow to calculate hadronic form factors:
several important applications to exclusive B decays

based on:

- vacuum $\longrightarrow$ hadron correlators
- OPE near the light-cone,
- inputs: light-cone distribution amplitudes of $\pi, K, \rho, ...$
- dispersion relations $\oplus$ parton-hadron duality

"remake" of QCD sum rules: Shifman, Vainshtein, Zakharov (1979)
based on vacuum $\rightarrow$ vacuum correlators, OPE in local operators ,
inputs: quark/gluon condensates

LCSR for $B \to \pi$ form factor



The correlation function: $q^2, (p+q)^2 \ll m_b^2$, b-quark highly virtual

$$F_\lambda(q,p) = i \int d^4x e^{iqx} \langle \pi(p) \mid T\{\bar{u}(x)\gamma_\lambda b(x), \bar{b}(0) i\gamma_5 d(0)\} \mid 0\rangle$$

operator-product-expansion (OPE) near the light-cone, $x^2 \sim 0$

OPE near the light-cone

schematically,

$$F^{(B)}(q,p) = i\int d^4x e^{iqx}\Big\{ \big[S_0(x^2,m_b^2,\mu) + \alpha_s S_1(x^2,m_b^2,\mu)\big] \otimes \langle \pi(p) \mid \bar{u}(x)\Gamma d(0)\,|0\rangle|_\mu$$

$$+ \int_0^1 dv\ \tilde{S}(x^2,m_b^2,\mu,v) \otimes \langle \pi(p) \mid \bar{u}(x)G(vx)\tilde{\Gamma} d(0)\} \mid 0\rangle|_\mu \Big\} + ...$$

* $S_{0,1}$, $\tilde{S}$ - perturbative amplitudes, (virtual b-quark)

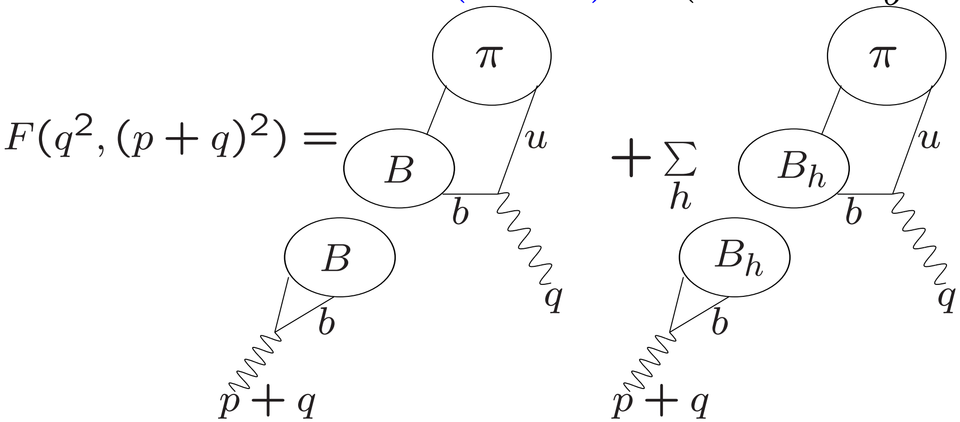
* universal distribution amplitudes of π (or K, ρ, K^*):

$$\langle \pi(q)|\bar{u}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2=0} = -iq_\mu f_\pi \int_0^1 du\, e^{iuqx}\varphi_\pi(u) + O(x^2)\ .$$

* the expansion (light-cone OPE) goes over $\alpha_s(\mu)$ and powers of $1/\mu^2$;
* typical scale $\mu^2 \sim m_b\Lambda$, where $\Lambda \sim 1$ GeV $\gg \Lambda_{QCD}$

Derivation of LCSR

- Hadronic dispersion relation in $(p+q)^2$: $(q^2 \ll m_b^2$ fixed)



$f_B f^+_{B\pi}(q^2)$ $\qquad\qquad$ $\sum_{B_h} \rightarrow$ *duality* (s_0^B)

- $[f_{B\pi}(q^2)]_{LCSR}$ includes *both* "soft" (end-point) and "hard" $(\sim \alpha_s)$ contributions, valid at $0 < q^2 < m_B^2 - \mu^2$

- more details/results: talk by Roman Zwicky

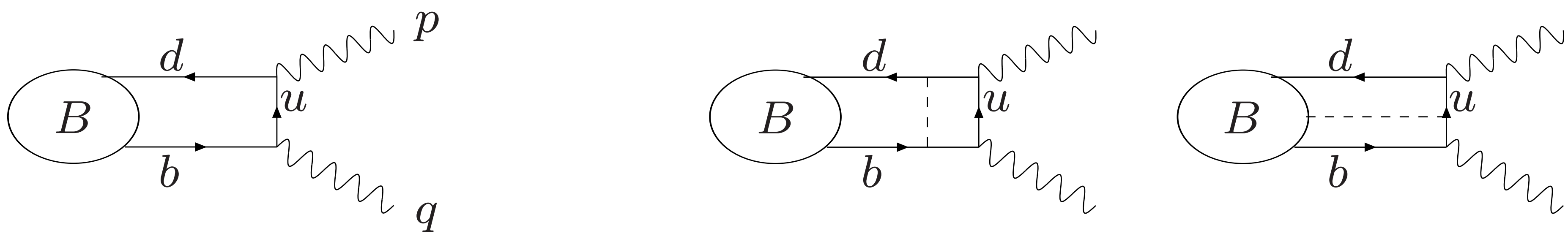
New approach: LCSR with B meson DA

A.K., T. Mannel, N.Offen PLB(2005), hep-ph/0504091
also (in SCET): F. De Fazio, T. Feldmann and T. Hurth; hep-ph/0504088

- The "inversed" correlator:

B meson on-shell, pion interpolated with an axial current $\oplus$ duality

$$F^{(B)}_{\mu\nu}(p,q) = i\int d^4x\; e^{ip\cdot x}\langle 0|T\left\{\bar{d}(x)\gamma_\mu\gamma_5 u(x), \bar{u}(0)\gamma_\nu b(0)\right\}|\bar{B}^0(p+q)\rangle\,.$$



$q^2=0$, $p^2<0$, $|p^2|\gg\Lambda^2_{QCD}$, u-quark propagates near LC .

OPE near the light-cone

schematically, (on-shell B meson with $v = p_B/m_b$)

$$F(q,p) = i \int d^4x e^{iqx} \Big\{ \left[S_0(x^2,\mu) + \alpha_s S_1(x^2,\mu)\right] \otimes \langle 0 \mid \bar{d}(x)\Gamma b(0) \mid B(v)\rangle|_\mu$$

$$+ \int_0^1 dv\ \tilde{S}(x^2, m_b^2, \mu, v) \otimes \langle 0 \mid \bar{d}(x)G(vx)\tilde{\Gamma} b(0)\} \mid B(v)\rangle|_\mu \Big\} + \ldots$$

* $S_{0,1}$, $\tilde{S}$ - perturbative amplitudes, (virtual u-quark)

* universal distribution amplitudes of $B(v)$: $\langle 0|\bar{d}(x)[x,0]\Gamma b(0)|B(v)\rangle_{x^2=0}$

- first uses of B-meson DA in PQCD factorization for $B \to \pi$

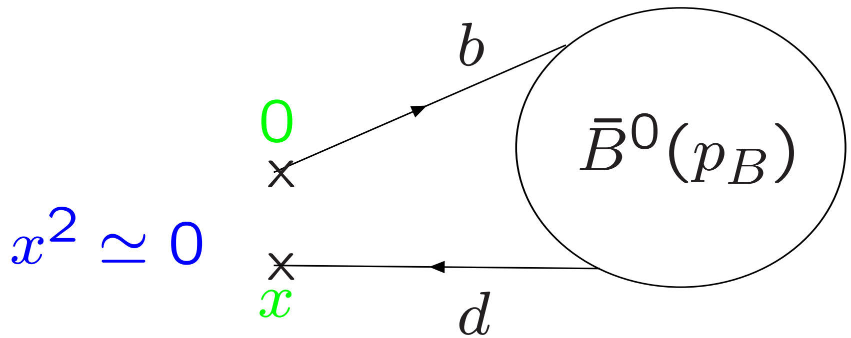
. A. Szczepaniak, E. M. Henley and S. J. Brodsky (1990)

. R. Akhoury, G. Sterman and Y. P. Yao (1994)

B-Meson two-particle DA: the definition

.

A.G.Grozin. M.Neubert (1997)



- Light-cone matrix element, consistent with HQET

$$\langle 0|T\left\{\bar{d}_\alpha(x)[x,0]b_\beta(0)\right\}|\bar{B}^0(v)\rangle|_{x^2=0}$$

$$= -\frac{if_Bm_B}{4}\left[(1+\not{v})\gamma_5\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left\{\phi_+^B(\omega)+\frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha},$$

$[x,0]$-Wilson line, two normalized DA's $\phi_+^B(\omega)$ and $\phi_-^B(\omega)$,

- variable $\omega=(l_0+l_3)$: (l-light spectator momentum in B rest frame)

Factorization in $B \to \gamma l \nu_l$

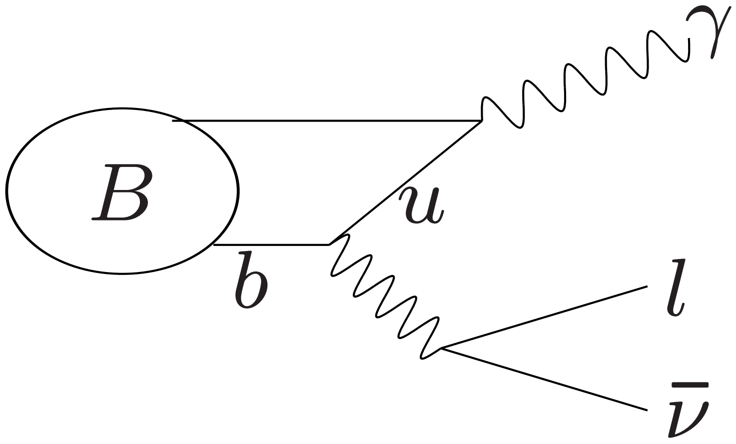
$(p_l + p_\nu)^2 \sim 0, \; E_\gamma \sim m_B/2$

$A(B \to \gamma l \nu) \sim \int d\omega \phi_+^B(\omega) T_h(\omega)$

$T_h \sim 1/\omega,$

$1/\lambda_B = \int_0^\infty d\omega \frac{\phi_+^B(\omega)}{\omega}$

- the inverse moment



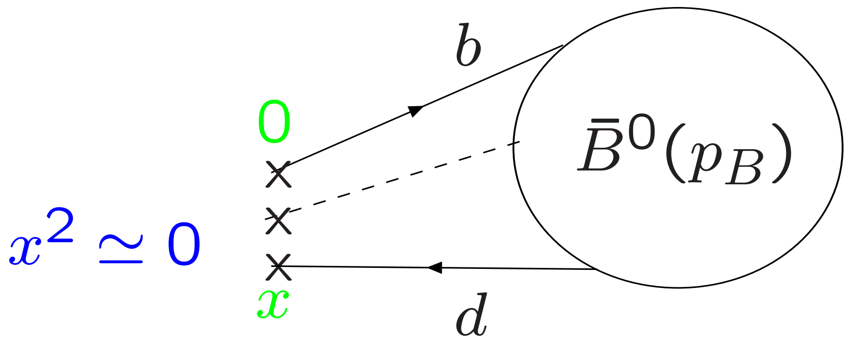
G. P. Korchemsky, D. Pirjol, T. M. Yan (2000)
S. Descotes-Genon, C. T. Sachrajda (2003)]
S. W. Bosch, R. J. Hill, B. O. Lange, M. Neubert (2004)

- factorization in $B \to \pi$, $B \to h_1 h_2$ etc.

Quark-antiquark-gluon DA's: definition

.
.

H. Kawamura, J. Kodaira, C.F.Qiao and K. Tanaka,(2001)



$$\langle 0|\bar{d}_\alpha(x)G_{\lambda\rho}(ux)b_\beta(0)|\bar{B}^0(v)\rangle = \frac{f_B m_B}{4}\int_0^\infty d\omega \int_0^\infty d\xi\, e^{-i(\omega+u\xi)v\cdot x}$$

$$\times\Bigg[(1+\not{v})\Bigg\{(v_\lambda\gamma_\rho - v_\rho\gamma_\lambda)\Big(\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)\Big) - i\sigma_{\lambda\rho}\Psi_V(\omega,\xi)$$

$$-\left(\frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x}\right)X_A(\omega,\xi) + \left(\frac{x_\lambda\gamma_\rho - x_\rho\gamma_\lambda}{v\cdot x}\right)Y_A(\omega,\xi)\Bigg\}\Bigg]_{\beta\alpha}.$$

What do we know about B-meson DA's

• model-independent relations from QCD equation of motion, e.g. Wandzura-Wilczek-type:

$$\phi_-^B(\omega) = \int_\omega^\infty d\rho \frac{\phi_+^B(\rho)}{\rho} + \int d\omega d\xi \{\Psi_{V,A}(\omega,\xi)\} \quad \Rightarrow \phi_-^B(0) = 1/\lambda_B + \{corr.\}$$

• boundary conditions: $\omega \to 0$: $\phi_+^B(\omega) \sim \omega, \quad \phi_-^B(0) = const$

• Evolution of $\phi_+^B(\omega,\mu)$ calculated in HQET is nontrivial,
$\Rightarrow \lim_{\omega\to\infty} \phi(\omega) \sim -\log(\omega/\mu)/\omega$, "radiative tail" [M. Neubert, B. Lange, (2003)]

• no parton interpretation, positive moments divergent,
but $\lambda_B(\mu)$ well defined in $O(\alpha_s)$

• no problem for the new sum rules containing integrals over small $\omega < s_0/m_B$

- models of $\phi_{\pm}^{B}(\omega)$ based on QCD sum rules in HQET

[A. G. Grozin and M. Neubert (1997)]

- The correlator for $\phi_{+}^{B}(\omega)$:

$$i\int d^4x\, e^{-ik(vx)}\langle 0|\mathrm{T}\{O_+(t)\bar{h}_v(x)\Gamma_2 q(x)\}|0\rangle = \{...\}T(t,k)\,.$$

$O_+(t) = \bar{q}(tn)\, \not{n}\,[tn,0]\Gamma\, h_v(0),$

$k < 0$ - external (Euclidean) momentum variable,

$k = \bar{\Lambda}$ is B meson pole in HQET , $\{...\}$ - a trace

- loop $\oplus$ condensate $\Rightarrow$ simple ansatz

$$\phi_{+}^{B}(\omega) = (\omega/\omega_0^2)e^{(-\omega/\omega_0)}, \quad \phi_{-}^{B}(\omega) = (1/\omega_0)e^{(-\omega/\omega_0)},$$

- A hybrid model for $\phi(\omega,\mu)$ (exponent.ansatz with the radiative tail)

S.J. Lee, M. Neubert, (2005)

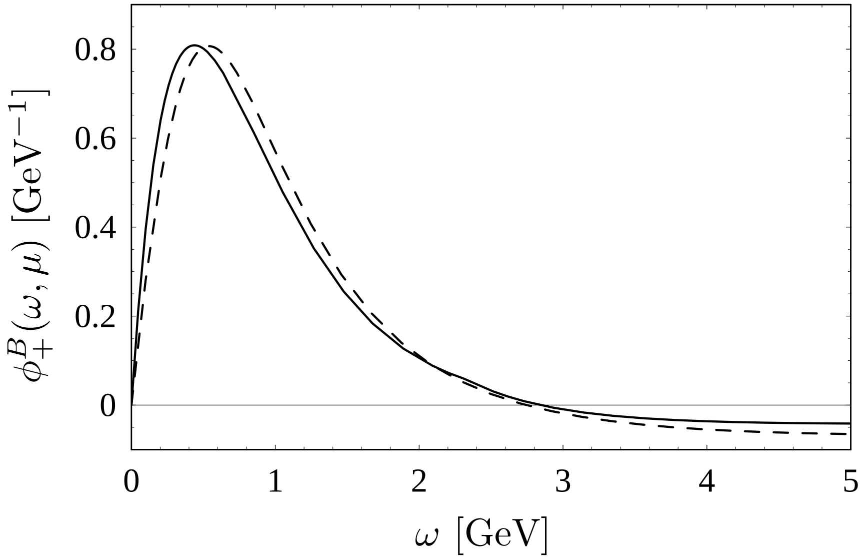
- NLO calculation (including radiative corrections)

[V. M. Braun, D. Y. Ivanov and G. P. Korchemsky,(2003)]

- the sum rule fitted to an explicit ansatz for $\varphi_+^B(\omega)$,

$$\phi_+^B(\omega, \mu = 1 \text{ GeV}) = \frac{4\lambda_B^{-1}}{\pi} \frac{\omega}{\omega^2+1}\left[\frac{1}{\omega^2+1} - \frac{2(\sigma_B - 1)}{\pi^2} \ln\omega\right],$$

(ω in units of GeV) $\lambda_B = (460 \pm 110)$MeV, $\sigma_B = 1.4 \pm 0.4$ at $\mu = 1$ GeV



solid (dashed) is the Lee-Neubert (Braun-Ivanov-Korchemsky)

Deriving the simplest sum rule

- OPE result for $B \to \pi$, the LO diagram: only $\phi_-^B(\omega)$ contributes

$$F_{\mu\nu}^{(B)} = 2if_B \int\limits_0^\infty \frac{d\omega}{m_B\omega - p^2}\phi_-^B(\omega)p_\mu p_\nu + \dots ,$$

- Hadronic dispersion relation:

$$F_{\mu\nu}^{(B)} = \langle 0|\bar{d}\gamma_\mu u|\pi(p)\rangle\langle\pi(p)|\bar{u}\gamma_\nu b|B(p+q)\rangle + \dots$$
$$= \left\{ \frac{2if_\pi f_{B\pi}^+(0)}{-p^2} + \int\limits_{s_h}^\infty ds \frac{\rho^h(s)}{s-p^2} \right\} p_\mu p_\nu + \dots ,$$

apply duality in pion channel⊕ Borel transformation.

• The relation between B meson parameters: (using $s_\pi^0 \ll m_B^2$):

$$\frac{1}{\lambda_B} \simeq \frac{f_\pi f^+_{B\pi}(0) m_B}{f_B M^2 (1 - e^{s_0^\pi / M^2})} \,.$$

• inputs: use LCSR for $B \to \pi$ form factor (in terms of pion DA's), 2pt sum rule for f_B and predict λ_B

• 3-particle B meson DA's, enter
1) soft-gluon diagram
2) indirectly, violation of WW relation estimated - a few %

Summary on the inverse moment

$1/\lambda_B = \int_0^\infty d\omega \frac{\phi_+^B(\omega)}{\omega}$ renorm. scale $\sim$ 1 GeV

Method	λ_B [MeV]	Ref.
2pt SR in HQET,LO	$\simeq 350$	Grozin,Neubert
2pt SR in HQET, NLO	440 ± 110	Braun, Ivanov,Korchemsky
LCSR for $B \to \gamma l \nu_l$	$\simeq 600$	Ball, Kou
"inverted" LCSR for $B \to \pi$	460 ± 160	A.K.,Mannel, Offen
first moments +Ansatz	480 ± 55	Lee, Neubert

- the new method allows to calculate many different $B \to$ light form factors "in one go", including SU(3) breaking, $m_q = m_s$

- the main advantage: knowledge of pion, K, ρ, K^* DA's not needed, decay constants, duality thresholds from exp. and/or two-point (SVZ) SR

- contributions of 3-particle DA's of B meson suppressed by powers of $\sqrt{s_0}/m_B$ (work in progress) ,

- preliminary (sample) results for all major heavy-light form factors:

  inputs: 2-particle DA,s $\phi_\pm^B(\omega)$, Grozin-Neubert exponential ansatz, $\lambda_B = 440$ MeV, $f_B = 180$ MeV, $M^2 = 1$ GeV, $m_s(1GeV) = 130MeV$ more detailed analysis in progress

- compared with the results of conventional LCSR [BZ] by P. Ball, R. Zwicky, (2005) ($B \to \pi, K, \rho, K^*$)

Form Factor	LCSR with B DA'(prelim)	LCSR with light-meson DA's	Ref.
$f^{+}_{B\pi}(0)$	$0.267\pm$	0.258 ± 0.03 0.26 ± 0.05 0.25 ± 0.05	[BZ] [KMMM] [AGRS]
$f^{+}_{BK}(0)$	$0.328\pm$	0.301 ± 0.041	[BZ]
$f^{T}_{B\pi}(0)$	$0.24\pm$	0.253 ± 0.028	[BZ]
$f^{T}_{BK}(0)$	$0.305\pm$	0.328 ± 0.04	[BZ]
$V^{B\rho}(0)$	$0.382\pm$	0.323 ± 0.029	[BZ]
$V^{BK^*}(0)$	$0.442\pm$	0.411 ± 0.033	[BZ]
$A_1^{B\rho}(0)$	$0.281\pm$	0.242 ± 0.024	[BZ]
$A_1^{BK^*}(0)$	$0.328\pm$	0.292 ± 0.028	[BZ]
$A_2^{B\rho}(0)$	$0.253\pm$	0.221 ± 0.023	[BZ]
$A_2^{BK^*}(0)$	$0.304\pm$	0.259 ± 0.027	[BZ]
$T_1^{B\rho}(0)$	$0.323\pm$	0.267 ± 0.021	[BZ]
$T_1^{BK^*}(0)$	$0.375\pm$	0.333 ± 0.028	[BZ]

*[KMMM] A. K.,T. Mannel, M. Melcher and B. Melic, PRD (2005), hep-ph/0509049

*[AGRS] Arnesen, Grinstein, Rothstein, Stuart, hep-ph/0504209

Conclusions

- new type of LCSR , calculating $B \to \text{light}$ form factors:
already leading order has a good agreement with LCSR with light meson DA's

- quantitative estimates of SU(3) breaking effects;

- sensitivity to the inverse moment λ_B, values $< 300 MeV$ disfavored