Recent results in Color Superconductivity

G. Nardulli

QCD Phase at T=0, large μ

Gluonic interaction between two quarks

$$3 \times 3 = \bar{3} + 6$$

The color antisymmetric $\bar{3}$ channel is attractive. Therefore Cooper's theorem implies formation of a condensate of Cooper pairs of quarks (diquarks) and color superconductivity

$$\langle \psi\psi \rangle = \Delta \neq 0$$

Color Superconductivity at extreme densities

- **S=0** state is preferred as more phase space available
- Pauli principle implies antisymmetry in flavor
- Asymptotically (very large densities):Color Flavor Locking (CFL; Alford, Rajagopal, Wilczek)

$$< 0 \mid \psi^i_{\alpha} \, \psi^j_{\beta} \mid 0> = \Delta \, \varepsilon^{ij\gamma} \, \varepsilon_{\alpha\beta\gamma}$$

Valid for $\mu >> m_q$

Rigourously proved in QCD, albeit at very large μ.

At intermediate densities, relevant for astrophysical applications the ground state is still unknown

Model calculations (NJL) as QCD unreliable

Deviations from CFL due to s quark mass effect.

Color and electric neutrality must be imposed;

=> different gaps for quarks of different flavor.

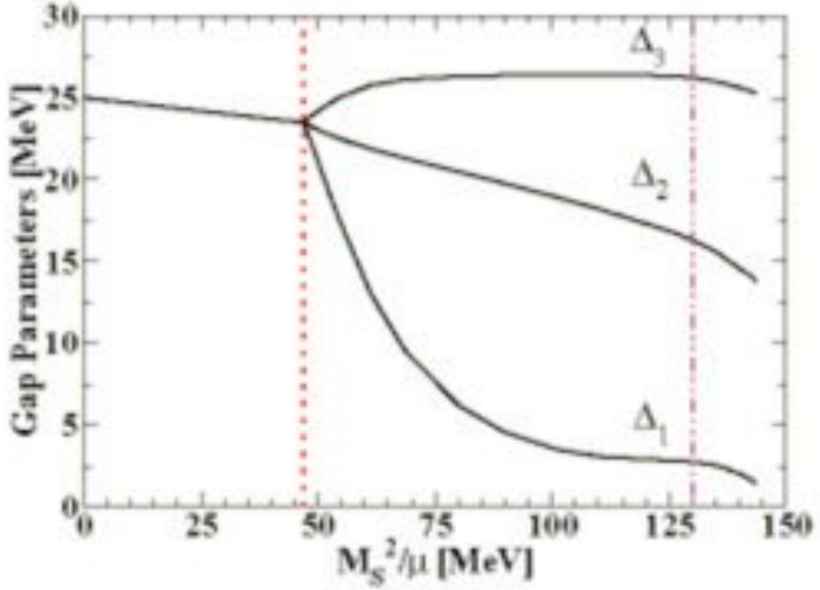
Gapless phases uniform (gCFL) or not uniform (LOFF).

Choosing the true vacuum

The true ground state has the minimum value of the free energy (grand potential)

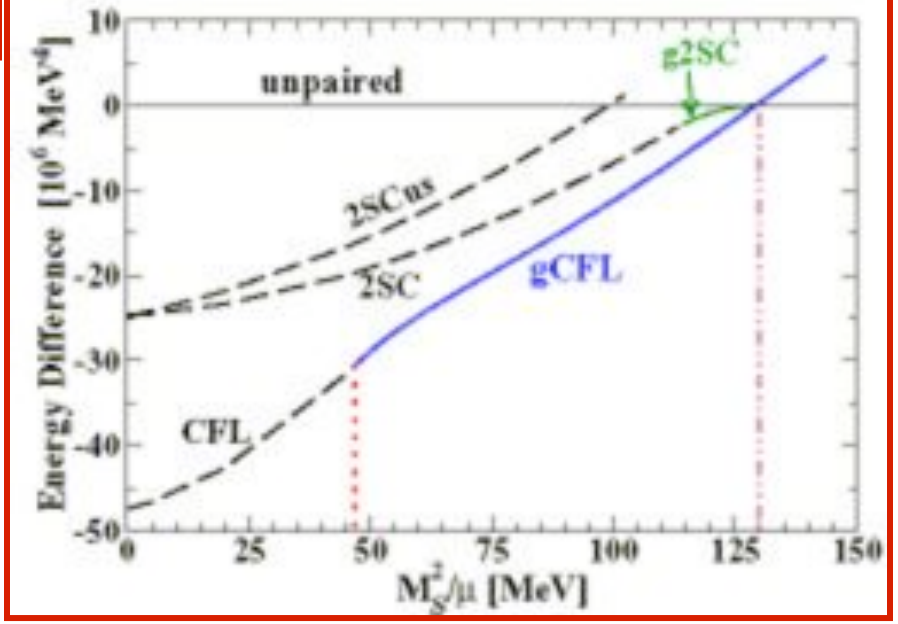
$$\Omega = -\frac{1}{2\pi^2}\int_0^\Lambda dp\, p^2 \sum_{j=1}^{9} |\epsilon_j(\ell_\parallel)| + \frac{1}{G}(\Delta_1^2+\Delta_2^2+\Delta_3^2) - \frac{\mu_Q^4}{12\pi^2}$$

Results for gCFL phase



Gap Parameters

Free energy



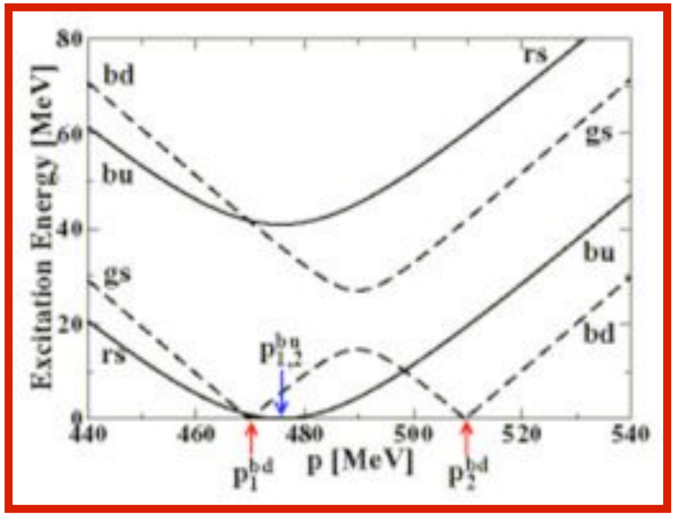
M.Alford, P.Jotwani, C. Kouvaris, J. Kundu, K.Rajagopal

The importance of being gapless

μ= 500 MeV
Δ= 25 MeV
$M^2_s/2\mu$= 80 MeV

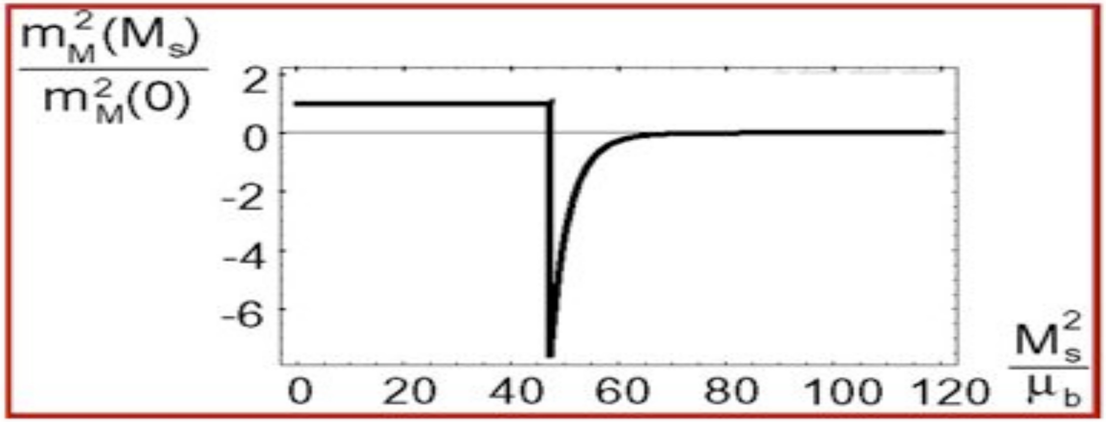
$$|\epsilon(p)| = |\delta\mu \pm \sqrt{(p-\mu)^2 + \Delta^2}|$$

$$p_{gapless} = \mu \pm \sqrt{\delta\mu^2 - \Delta^2}$$



Gapless modes: relevant for astrophysical effects (phenomenology of compact stars)

Meissner masses in the gCFL phase



Solid line a=1,2;
One Gluon"3mixed8":
same behavior;
others: no instability
One $U_{em}(1)$: massless

R. Casalbuoni, M. Mannarelli, G. N., M. Ruggieri, R. Gatto

Imaginary masses: instability of chromomagnetic type

gCFL is not the true vacuum

Similar results for 5 gluons of broken generators in 2-flavor (Huang-Shovkovy)

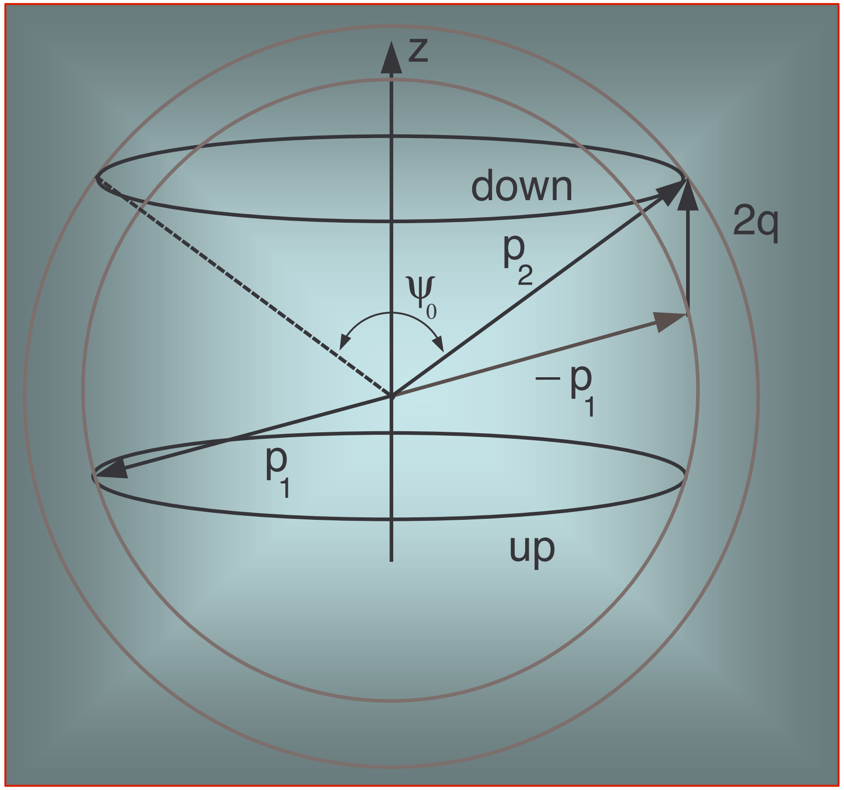
LOFF phase favored (Giannakis-Ren, Fukushima)

Loff Phase: two flavors (u,d)

If $\delta\mu\neq0$ it can be energetically favourable to have states where the Cooper pair has total momentum $2q \neq 0$

LOFF= Larkin-Ovchinnikov& Fulde-Ferrell (1964); Alford, Bowers, Rajagopal (2000) R. Casalbuoni, G.N. Rev. Mod. Phys.(2004)

Studied in condensed matter (2D superconductors), cuprates



$$\Delta(r) = <0 | \psi(r)\psi(r)|0> = \Delta \exp[i(p_1 + p_2)r]$$

LOFF phase: three flavors

Needed for realistic calculations

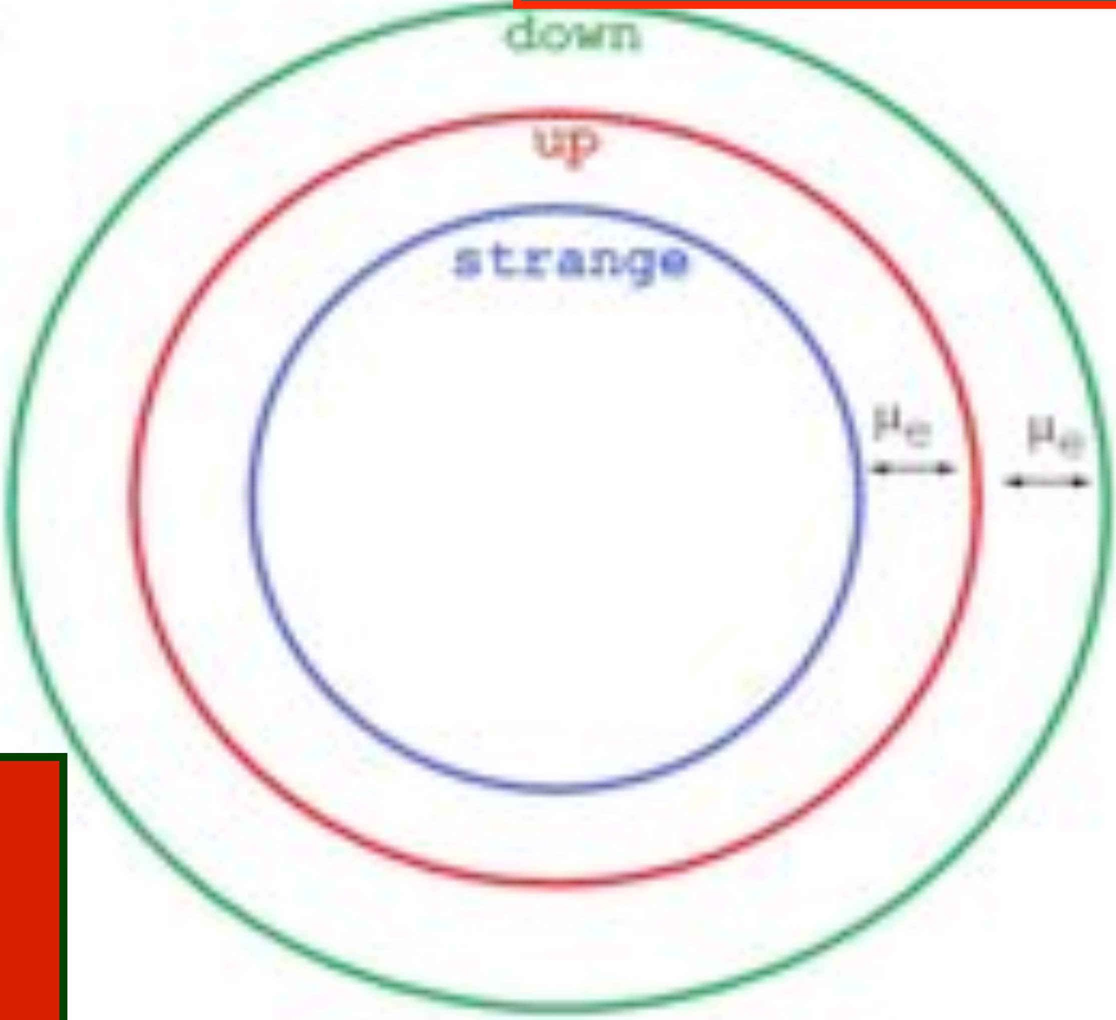
- **R. Casalbuoni, R.Gatto,,N.Ippolito,GN, M.Ruggieri : Ginzburg Landau expansion of the gap equation and free energy**
- **(Almost) complete analysis beyond GL: M.Mannarelli, K. Rajagopal, R. Sharma**

$\mu_e = \mu_d - \mu_u = \mu_u - \mu_s \sim M^2_s \; /4\mu$

$\mu_d - \mu_s = -2\,\mu_e$

Near the transition point du,us couplings, equal strength, no ds pairing

Fermisurfaces for quark matter in the normal phase, β equilibrium and non zero strange quark mass



Expected:

$\mu_e = m^2_s/4\mu$

$\Delta_1 = 0;\ \Delta_2 = \Delta_3$

Gap parameters

$$\Delta_K(\mathbf{r}) = \Delta_K\, e^{2i\,\mathbf{q}_k \cdot \mathbf{r}}$$

For each inhomogeneous pairing a Fulde-Ferrell ansatz;

$$2\;\mathbf{q}_k$$

represents the momentum of the Cooper pair.

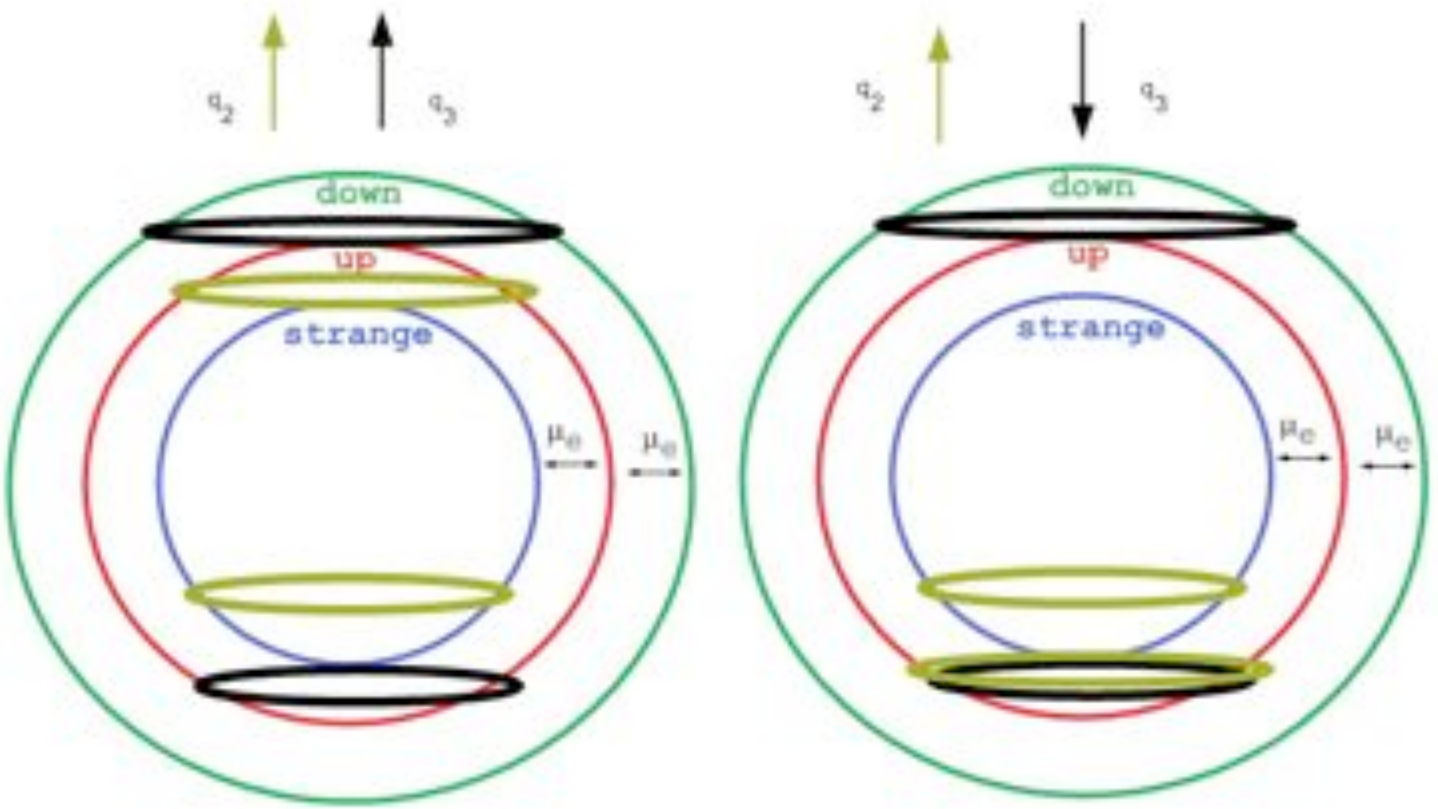
This is the simplest ansatz, other structures should be examined

Three independent functions

$\Delta_1(\mathbf{r})$, $\Delta_2(\mathbf{r})$, $\Delta_3(\mathbf{r})$,

describing respectively d-s, u-s and u-d pairing.

q_2 q_3 parallel favored by phase space



$q \simeq 1.2\ \delta\mu$

Ginzburg-Landau expansion

$$\Omega = \Omega_n + 1/4 \sum_{I=1}^{3} (2\alpha_I \Delta_I^2 + \beta_I \Delta_I^4 + \sum_{J\neq I} \beta_{IJ} \Delta_I^2 \Delta_J^2) + O(\Delta^6)$$

$$\Omega_n = -3/12\pi^2(\mu_u^4+\mu_d^4+\mu_s^4) - 1/\,12\pi^2\,\mu_e^4$$

R. Casalbuoni, R.Gatto,, N.Ippolito,GN, M.Ruggieri

$-\partial_{\Delta_k}\Omega = 0$ minimum in Δ_κ

$-\partial_{\mu_e}\Omega = 0$ electrical neutrality

Approximation: $\mu_3 \simeq \mu_8 \simeq 0$ (confirmed by R.Ciminale et al., in preparation: Valid near the transition point, since this is the result for the normal phase)

The numerical results in this regime confirm **$\Delta_1=0$; $\Delta_2 \simeq \Delta_3$**

(Almost) complete calculation

Assume from the very beginning

$$\Delta_1 = 0 \; ; \qquad \Delta_2 = \Delta_3$$

$$\mu_3 = \mu_8 = 0 \; ; \qquad \mu_e = m_s^2 / 4\mu$$

Since

$$k_u + k_d = 2q_3 \; ; \; k_u + k_s = 2q_2$$

Shift momenta so that

$$P_u = p = \mu\, v; \; P_d = p + 2q_3; \; P_s = p + 2q_2$$

Corresponding to redefinitions of $\delta\mu$:

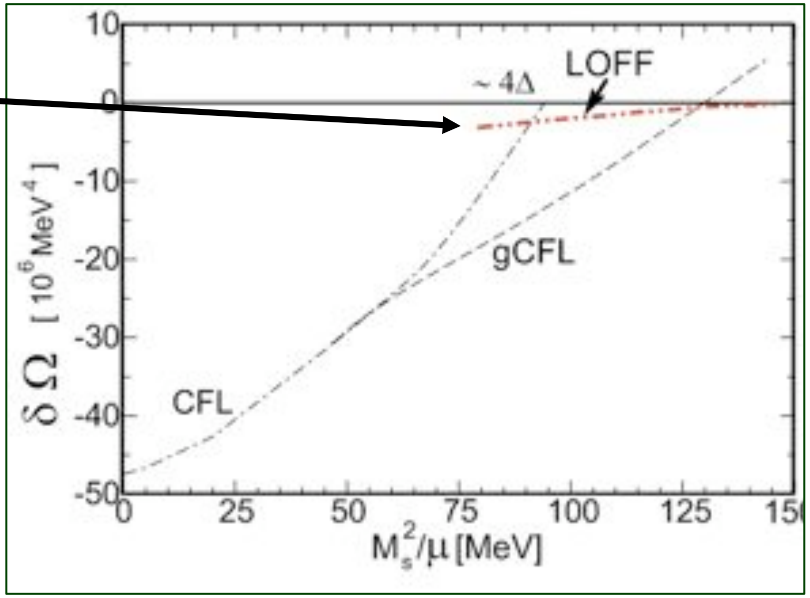
$$\delta\mu \to \delta\mu \pm q\, v$$

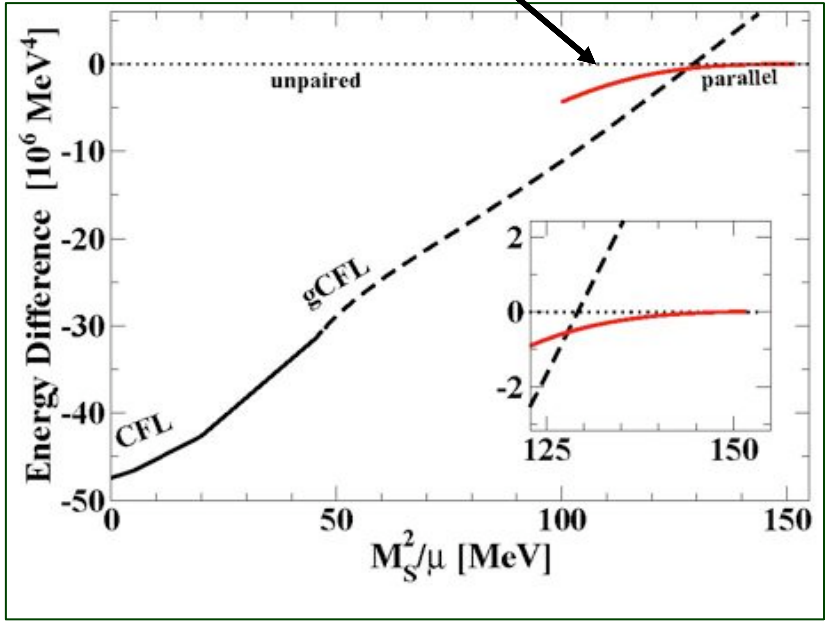
M.Mannarelli,
K. Rajagopal,
R. Sharma

Comparison among different CSC phases

Ginzburg Landau Approximation

Calculation at
leading order in μ

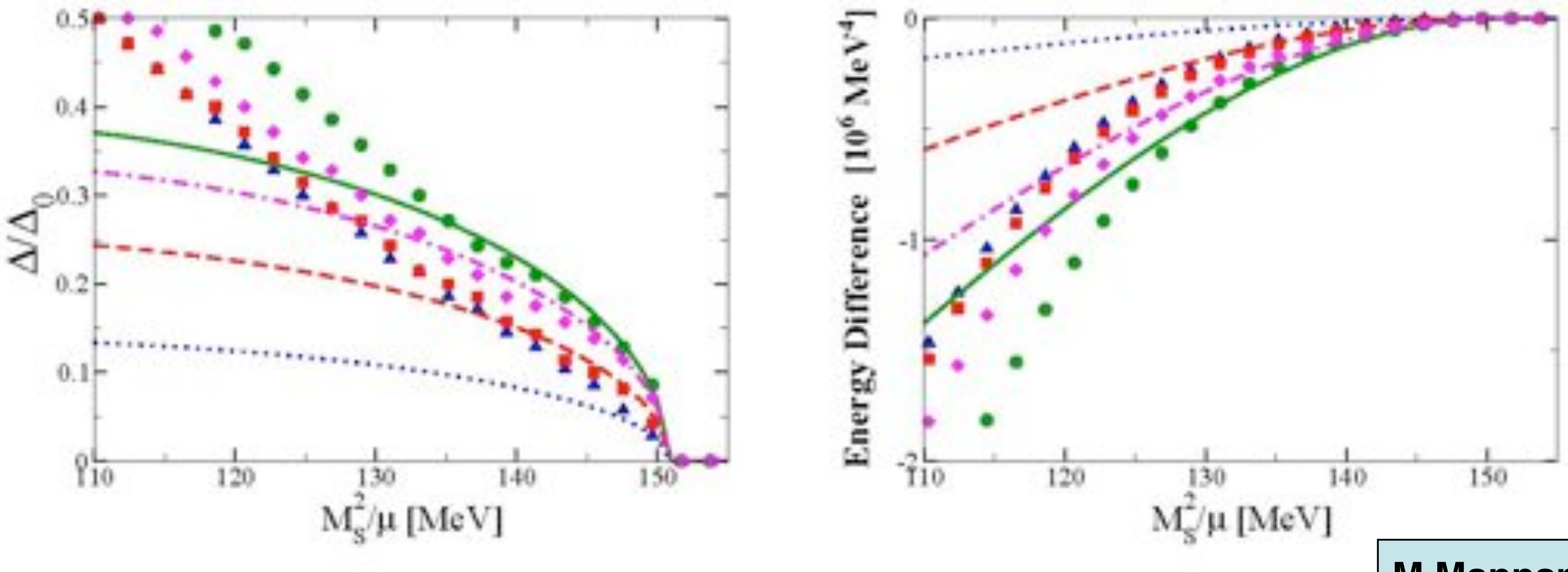




M.Mannarelli, K. Rajagopal, R. Sharma

R.Gatto, R.Casalbuoni, N.Ippolito, G.Nardulli, M.Ruggieri

Comparison between the two approaches



M.Mannarelli, K. Rajagopal, R. Sharma

Lines: GL. Dots: complete

Green= q_2 q_3 parallel. Magenta: 2/3π.

Red; 7/8π. Blue: 31/32 π

Chromomagnetic instability

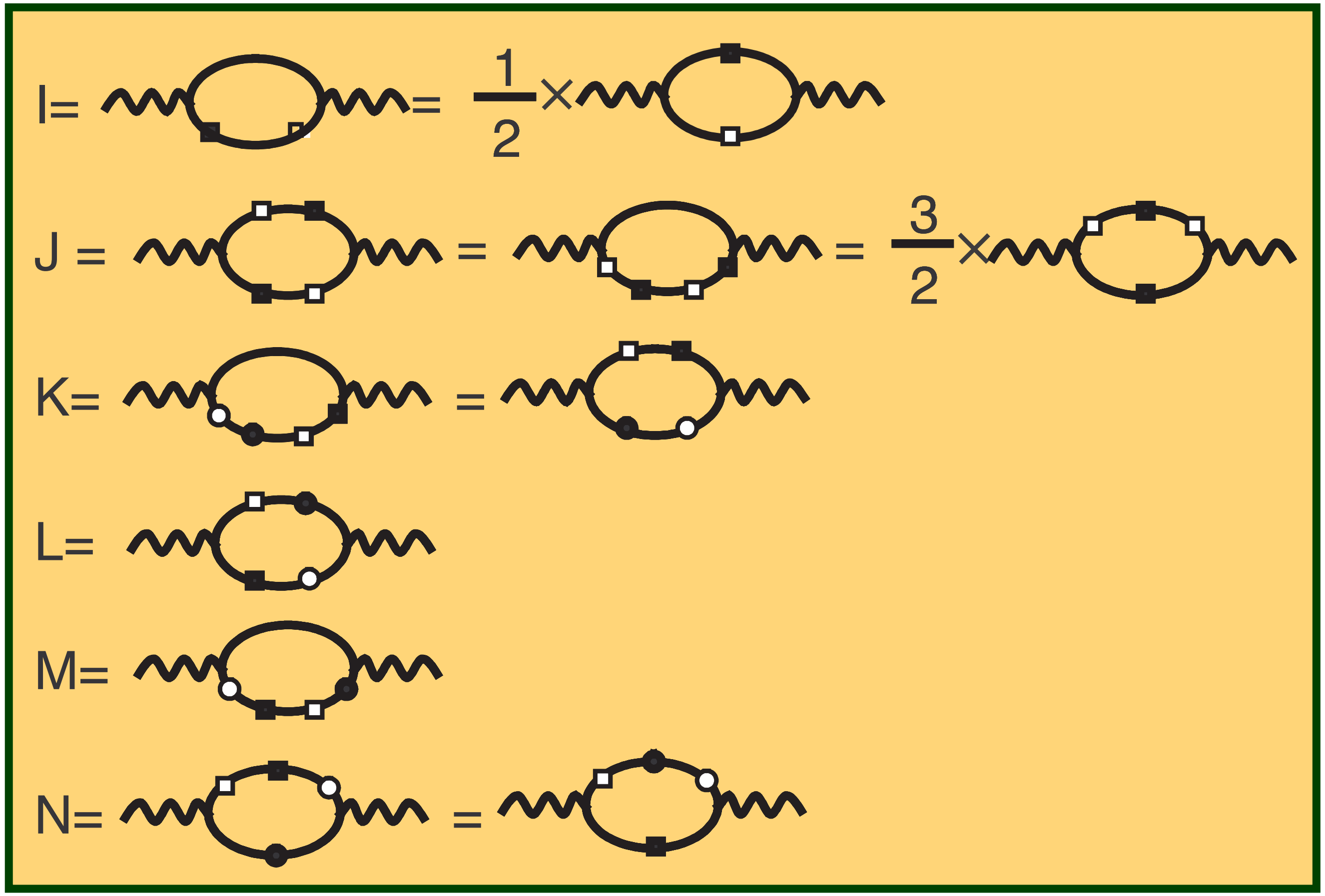
In gCFL : imaginary gluon Meissner masses

Studied in LOFF model with 2 flavors (Giannakis&Ren, Fukushima, Gorbar, Hashimoto&Miransky).

LOFF favored in comparison to gapless uniform conductive states. At least in the GL region (small gap) no chromomagnetic instability

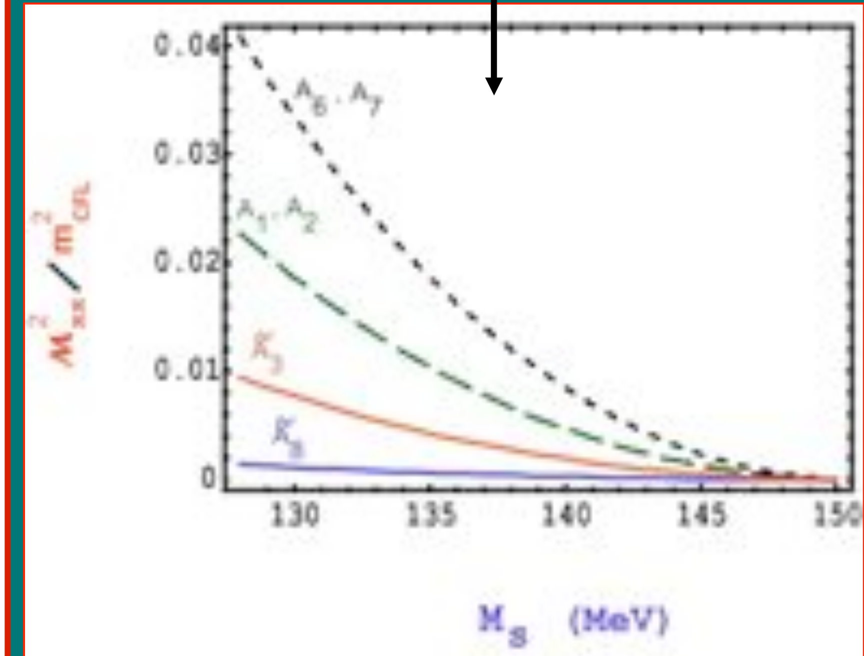
For 3 flavors: recent results (Ciminale,Gatto,GN,Ruggieri)

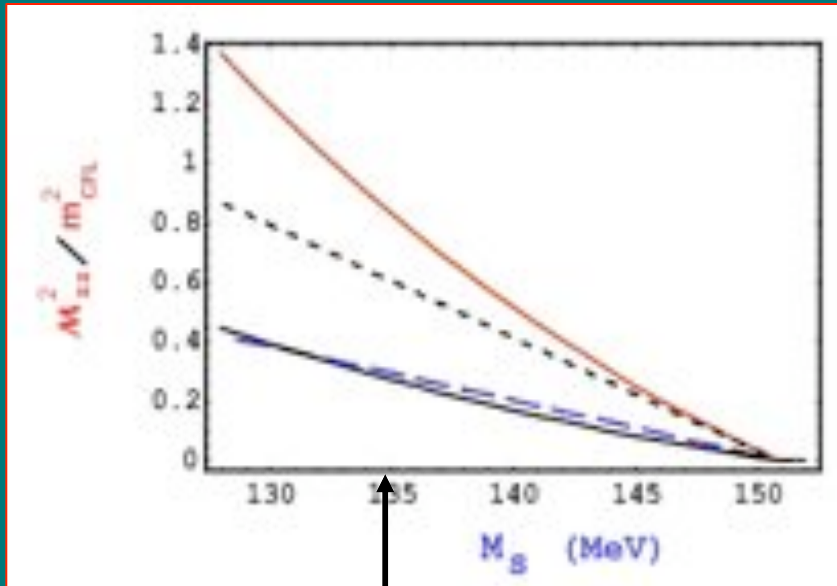
Feynman Diagrams at $O(\Delta^4)$



Gluon Meissner masses:no instability

Transverse mass





Longitudinal mass

M.Ciminale, R.Gatto,GN, M.Ruggieri

Conclusions

- QCD at intermediate densities, small T : active field of research
- Structure of vacuum not yet understood, though color superconductivity expected
- Likely gapless superconductivity, e.g. LOFF anisotropic state
- Results interesting for the effects on cooling of compact stars with a quark core: faster cooling